SEMINARIO DE ECONOMÍA

Xoves, 18 de maio de 2017

Título:

"Public debt expansions and the dynamics of the household borrowing constraint"

Ponente:

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Lugar e hora:

Aula-Seminario 6

15:00 h

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Public debt expansions and the dynamics of the household borrowing constraint*

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February, 2017

Abstract

We show that the endogeneity of the household borrowing constraint accounts for a sizable part of the effects in output, credit and welfare of fiscal policies that entail government debt expansions, within an incomplete-markets model featuring heterogeneous households. These policies make the borrowing constraint tighter because of a higher interest rate. The tightening favors a deleveraging process in terms of private credit and reinforces the precautionary saving motive. This in turn exerts a downward pressure on the interest rate, dampening the tightening itself. As an example, under a plausible debt-financed transfers policy, the majority of households supports the policy within our baseline economy with the endogenous borrowing constraint, whereas it is against the policy if such endogeneity is not considered.

JEL classification: E21, E44, E62, H60.

Keywords: Endogenous borrowing constraint, government debt, spending policies, heterogeneous households, precautionary saving motive.

*An earlier version of this paper, dated May 2015, circulated under the title “Debt-financed fiscal policies and the dynamics of the household borrowing constraint”. We thank Pedro Teles for several discussions at different stages of the project. We are also grateful to Mark Aguiar, Pedro Amaral, George-Marios Angeletos, Marco Bassetto, Pedro Brinca, Craig Burnside, Francesco Carli, Dean Corbae, Isabel Correia, Luca dei Federichi, Giulia Feserini, Jonathan Heathcote, João Victor Issler, Pat Kehoe, Tim Kehoe, Jesper Lindé, Fabrizio Mattesini, Ellen McGrattan, Ettore Panetti, Morten Ravn, Sérgio Rebelo, João Valle e Azevedo, and Gianluca Violante for useful comments and suggestions. We thank participants to all seminars and conferences in which this paper was presented. The views expressed are those of the authors and do not necessarily represent those of Banco de Portugal, Banca d’Italia or the Eurosystem. All errors are ours.

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1 Introduction

Policies have the potential to influence credit markets and, in particular, the households' ability to either borrow or lend. In this paper, we explicitly consider the endogeneity of the household borrowing constraint and show that this channel accounts for a sizeable part of the effects in output, credit and welfare of typical fiscal policies entailing public debt expansions.

It is well known that government debt expansions significantly influence the households' financial conditions (Woodford 1990, Aiyagari and McGrattan 1998, Angeletos et al. 2016, among others).[1] For example, Aiyagari and McGrattan (1998) put forward the view that, within an economy where households face borrowing constraints and the precautionary saving motive is active, public debt can act as if it loosened the household borrowing constraint. That is, higher levels of public debt result in higher interest rates, making assets more attractive to hold and, hence, enhancing households' self-insurance possibilities.

An increase in the interest rate contributes to a "loosening" of the borrowing limit, but it also makes borrowing more costly, generating *ceteris paribus* an actual tightening in the borrowing constraint. Virtually any endogenous borrowing constraint has the property of being proportional to the inverse of the borrowing cost, as are the cases of the natural borrowing limit in Aiyagari (1994) and of the constraint with collateral in Kiyotaki and Moore (1997). This property is supported by empirical evidence: a significant correlation between proxies for borrowing constraints, such as credit standards, and interest rates is documented in the data (Maddaloni and Peydró 2011).

We perform our analysis within a general equilibrium, flexible-prices, incomplete-markets model with physical capital which relies on the early contribution of Bewley (1977). Households are heterogeneous in terms of wealth, whose distribution is endogenous. In order to self-insure against the occurrence of bad productivity shocks, they borrow or lend without using collateral. We endogeneize the borrowing constraint by considering limited commitment for the repayment obligations of the households. In particular, in case of default they are permanently excluded from intertemporal trade, thus entering an autarky regime. We assume that honoring its own debt is at least as good as defaulting. Within our incomplete-markets environment and consistently with the empirical evidence, the temptation of declaring bankruptcy—measured by the relative value of autarky vis-à-vis the value of honoring debt commitments—decreases as the household's labor income increases.[2]

We calibrate the stationary distribution of our model at quarterly frequency for the U.S. economy. We then study the transition of the economy due to temporary public debt expansions that finance stylized but realistic spending policies. Our central analysis refers to the simplest policy one can think of: a debt expansion that finances transfers evenly distributed across

[1] Holmstrom and Tirole (1998) focus on the productive sector of the economy and study how the government-supplied assets influence the firms' financing problem.

[2] Kehoe and Levine (1993) and Alvarez and Jermann (2000) study the properties of the equilibrium allocation in models characterized by limited-commitment borrowing constraints, in the presence of a complete set of state-contingent securities. Furthermore, Zhang (1997), Ábrahám and Cárceles-Poveda (2010) and Antunes and Cavalcanti (2013) use these types of constraints within incomplete-markets models.

households. The aggregate profile for transfers follows the one estimated by Leeper et al. (2010). In the second experiment, the debt expansion finances an increase in purchases of goods and services similar to that set in the American Recovery and Reinvestment Act (ARRA). The two policies have a certain degree of persistence over time and entail future increases in taxation to repay the debt.

An increase in public debt positively impacts on the interest rate. All else equal, the option to stay in the market becomes relatively worse than going to autarky for the borrowers, giving them a higher incentive to default. Lenders will therefore be less willing to provide funds in the credit market. This will endogenously tighten the household borrowing constraint, meaning that the maximum quantity that households can borrow becomes smaller. Because of the tightening, constrained agents are forced to deleverage, while the unconstrained save more for precautionary purposes. The appetite for assets generates a downward pressure for the interest rate that dampens the above-mentioned tightening. On average, the tightening induces agents to cut consumption and work harder, though we show that the magnitude of the households' reactions depends on which region of the wealth distribution the household pertains to.

At the aggregate level, the debt expansion crowds out both credit and physical capital. The dynamics of the borrowing constraint, identified by its pure movement and price effects, explains a significant part of the households' reaction to the fiscal policies. For example, in the case of the transfers policy, over a five-year horizon the multiplier of our baseline model with the endogenous borrowing constraint is roughly -0.25, while the multiplier generated by the model in which the constraint is exogenously kept at its steady-state level is around -0.45. Regarding the credit dynamics at the same five-year horizon, roughly 20% of the fall in credit is explained by the tightening in both simulated policies.

The endogenous borrowing limit plays an important role in determining the welfare effects generated by the fiscal policies. In the case of the transfers policy, the dynamics of the borrowing limit crucially influences the political support to the policy. Within the baseline model, the majority of agents (roughly four-fifths) supports the policy, whereas in the fixed constraint version of the model such support is far from majoritarian. How can the support to such policy be larger in a situation where there is an endogenous tightening of the borrowing limit than in the case where the constraint is kept at its original level? Because the price effects induced by the dynamics of the borrowing limit along the transition—lower interest rates and higher wages—play a crucial role. They increase the utility of borrowers, wealth-poor and middle-class agents. They are instead detrimental for a small group of households, the wealth-rich, who mostly rely on asset income.

We also perform a crisis experiment by studying how the debt expansions influence the dynamics of an economy experiencing a credit crisis with both credit and output falling. The fiscal policies contribute to a further tightening and a more marked fall in credit. The effects on output are not substantial.

As mentioned above, our work is related to those papers that analyze the role of public debt within incomplete-markets models, like for example Woodford (1990), Aiyagari and McGrattan

(1998), Challe and Ragot (2011) and Angeletos et al. (2016). Our main contribution to this strand of the literature is to study how the actual dynamics of the household borrowing limit shapes the households' reaction and to provide a quantitative assessment. Our model belongs to the same framework as Aiyagari and McGrattan's; it is a heterogeneous agents model with incomplete insurance markets in which the agents' wealth distribution evolves endogenously. However, our work differs from theirs in several aspects. First, in the objective: they derive normative conclusions on the level of public debt, whereas we emphasize the endogeneity of the borrowing constraint and study how it influences the effects of the debt expansions. Second, in the modeling: they do not allow for private credit in the economy, and their main simulations are based on an exogenous credit limit that prevents households from borrowing. Third, in the analysis: theirs is based on comparisons between steady states while ours studies the transitional dynamics; we believe that analyzing the transition of the economy is appropriate when studying the effects of policies within a given country.

Regarding Woodford (1990), Challe and Ragot (2011) and Angeletos et al. (2016), they all use frameworks where the wealth distribution does not belong to the problem's state variables. We instead show that the household's reaction and welfare depend on its location within the endogenous wealth distribution. In particular, Angeletos et al. (2016) study the optimality of public debt in a Lagos and Wright (2005) type of model where this debt is used as collateral by private agents. They show that public debt can help relax the agents' collateral constraint, but it also tightens the government budget because of a higher interest rate. Differently from our work, they do not emphasize the effects of a higher interest rate on the private agents' borrowing limit.[3]

There is a recent stream of the literature that studies the effects of (i) taxes and monetary transfers and of (ii) government consumption within incomplete-markets frameworks. For example, Heathcote (2005), Ábrahám and Cárceles-Poveda (2010), Oh and Reis (2012), Kaplan and Violante (2014), Huntley and Michelangeli (2014) and McKay and Reis (2016) belong to the first class of papers, while Brinca et al. (2016) and Ercolani and Pavoni (2014) to the second one. Typically, these papers do not focus on the endogeneity of the borrowing constraints and on the role of public debt during the transitional dynamics. An exception is Ábrahám and Cárceles-Poveda (2010), who address the effects of revenue-neutral tax reforms within an economy which encompasses endogenous borrowing limits with limited commitment; they show that these borrowing constraints significantly influence the effects of the reforms.[4]

Our work is also related to those papers studying the effects of a credit crunch, within

[3]Woodford (1990) derives the optimal level of public debt within a deterministic model featuring liquidity constrained agents. Furthermore, Challe and Ragot (2011) study the effects of a government spending stimulus, through purchases, within a stochastic model where households face collateralized borrowing constraints. The authors mainly focus on the ability of the debt-financed spending shock to crowd in private consumption depending on the extent to which the fiscal policy enhances self-insurance possibilities. Their main simulations are based on an exogenous borrowing constraint set at zero; in a robustness exercise, they show that the increase in the interest rate can reduce the agents' ability to borrow.

[4]Notice that Ábrahám and Cárceles-Poveda (2010) use an analytical framework similar to ours but without considering an endogenous labor supply. Our results show that labor significantly interacts with the dynamics of the borrowing constraint.

frameworks of heterogeneous agents and incomplete markets, as, among others, Guerrieri and Lorenzoni (2016), Buera and Moll (2015) and Huo and Ríos-Rull (2015). Further, Kehoe et al. (2016) show that the joint dynamics of household debt and employment observed in the U.S. during the Great Recession is reproducible by fluctuations in the borrowing constraint within an otherwise standard incomplete-markets model with a housing good. We contribute to this literature by studying the interactions between the dynamics of the borrowing constraint and debt-financed fiscal policies.

Finally, there is a well-established stream of the literature that studies government spending stimuli within general equilibrium models, with complete markets and representative agent(s). The seminal contribution is represented by Baxter and King (1993). Furthermore, Galí et al. (2007), Hall (2009), Fernández-Villaverde (2010), Christiano et al. (2011), Eggertsson and Krugman (2012), Corsetti et al. (2013), Bilbiie et al. (2013) and Rendahl (2016) study the effects of fiscal policies in the presence of various financial frictions. Unlike them, our framework of analysis allows us to study how the combination of borrowing constraints, wealth heterogeneity and market incompleteness influences the households' reaction to public debt expansions.

The paper is structured as follows. Section 2 presents the model. Section 3 presents the results for the stationary distribution. Section 4 reports the transitional dynamics of the economy generated by the government debt expansions. Section 5 concludes.

2 Model

Our model belongs to the long-standing tradition of incomplete-markets models like, for example, Bewley (1977) and Aiyagari (1994). Specifically, we consider a general equilibrium model with capital and a neoclassical labor market in which households differ by their wealth and productivity. Households choose their level of consumption and of labor effort. They save or borrow using uncollateralized credit.

In the spirit of Kehoe and Levine (1993), Zhang (1997) and Alvarez and Jermann (2000), we endogeneize the borrowing constraint by allowing households to default on their debt, in which case they go to autarky permanently. Lenders will then make sure that the value of honoring debt by borrowers is not less than defaulting. Notice two important features of this type of constraints. First, unlike in a complete-markets setting (Kehoe and Levine 1993, Alvarez and Jermann 2000), and consistently with the data, using these constraints in an incomplete-markets framework makes the willingness of declaring default decreasing with the level of household's labor income. Second, unlike the standard natural borrowing limit of Aiyagari (1994), this type of constraint allows us to generate a realistic credit-to-output ratio and share of constrained households. The details of both characteristics are spelled out in Section 3.[5]

We also model a fiscal authority that can collect lump-sum, capital and labor taxes. It issues debt with the same return as physical capital to finance either transfers or purchases.

[5]Given the type of constraint used in the model, we cannot study the dynamics of households' default in equilibrium. Though we acknowledge that this channel can be potentially important when studying the movements of the borrowing limit, allowing for default goes beyond the objective of the paper.

Within our framework, the Ricardian equivalence does not hold even if the financing operates through a mix between debt and lump-sum taxation. This is because the class of borrowing constraint used in our model is tighter than the natural borrowing limit (see Chapther 9 of Ljungqvist and Sargent 2004, for a detailed argument).

2.1 Households and firms

There is a continuum of infinitely lived and *ex ante* identical households with measure one. We use a standard neoclassical instantaneous utility function of the following type:

$$u(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \chi\frac{n^{1+\psi}}{1+\psi}\,,$$

where c and n are consumption and labor, respectively. The individual state vector is defined as $x = (a, z)$, where a and z are asset holdings and productivity, respectively. Process z is finite-state Markov with support $\mathcal{Z}$ and transition probability matrix Π, whose element π_{ij} is defined as $\Pr(z' = z_j | z = z_i)$ and z_k is the k^{th} element of $\mathcal{Z}$. We shall henceforth use the usual notation where x' denotes the value of variable x in the next period.

The household problem in recursive form can be written as follows:

$$v(x,\theta) = \max_{c,n,a'} u(c,n) + \beta\mathbf{E}\left[v(x',\theta')|z\right] \tag{1}$$

subject to

$$c + a' = (1 + r(1-\tau_k\mathcal{I}_{a\geq 0}))a + wnz(1-\tau_w) + \mathrm{Tr} - \Gamma$$

$$v(x',\theta') \geq \underline{v}(z',\theta'), \quad \text{all } z' \in B(z) \tag{2}$$

$$\underline{v}(z,\theta) = \max_{n} u(\gamma\, wnz(1-\tau_w) + \mathrm{Tr} - \Gamma, n) + \beta\mathbf{E}\left[\underline{v}(z',\theta')|z\right] \tag{3}$$

$$\theta' = H(\theta)\,.$$

In these expressions, $\mathcal{I}_{a\geq 0}$ is an indicator function that equals 1 if $a \geq 0$, and 0 otherwise. $B(z) = \{\xi \in \mathcal{Z} : \Pi(z,\xi) > 0\}$ is the set of possible next-period idiosyncratic states given that the current state is z. θ is the measure of households, defined in a set of possible asset holdings and idiosyncratic shocks. It subsumes all relevant aggregate variables taken as given by the household. $H(\theta)$ is the forecasting function used by households in predicting next period's measure. Variable Tr represents transfers from the government to households, while Γ are lump-sum taxes. We need to distinguish among these two variables to allow for the coexistence of an exogenous policy of lump-sum transfers to households and a rule-based lump-sum tax. The net return on capital, or the borrowing cost, is r and the wage rate for labor efficiency units is w. Capital income is taxed at rate τ_k and labor income is taxed at rate τ_w.

Equation (2) is the individual rational constraint; it states that households have the option to go bankrupt. If so, they renege on all their existing debt and they are excluded from future participation in capital and credit markets; at the same time, their human capital is inalienable.

Equation (3) defines the value of being in autarky, $\underline{v}(z, \theta)$; in its expression, γ is one minus a pecuniary cost of having tainted credit status, as in Chatterjee et al. (2007). Notice that constraint (2) guarantees that it is never in the household's best interest to default. Since $v(x', \theta')$ is non decreasing in a' while $\underline{v}(z', \theta')$ is independent of a', equation (2) defines a set of endogenous lower bounds on borrowing conditional on each level of z'. Formally, we define $\hat{\underline{a}}(z', \theta')$ as the lowest, or most negative, possible asset level conditional on each level of z':

$$\hat{\underline{a}}(z', \theta') = \inf\{a' \in \mathbb{R} : v(a', z', \theta') \geq \underline{v}(z', \theta')\} . \tag{4}$$

For each current level of idiosyncratic productivity, z, the lender will pick the tightest constraint among those associated to next period's possible levels of productivity:

$$\underline{a}_z(\theta') = \sup\{\hat{\underline{a}}(z', \theta') : z' \in B(z)\} . \tag{5}$$

In practice, as we will see in Section 3, given the characteristics of our calibrated Π the relevant endogenous borrowing limit is unique and generated by inequality (2) parameterized in the lowest z, which corresponds to the tightest among the borrowing limits in (5).

A representative firm with production function $Y = AK^{\alpha}N^{1-\alpha}$ chooses efficient labor, N, and capital, K, taking factor prices as given, according to:

$$r^K = \alpha A \left(\frac{N}{K}\right)^{1-\alpha}, \text{ where } r = r^K - \delta \tag{6}$$

$$w = (1-\alpha) A \left(\frac{K}{N}\right)^{\alpha}, \tag{7}$$

where A is total factor productivity (TFP).

2.2 Government

We will assume a fiscal sector similar to Uhlig (2010). We consider the gap to finance in each period as the following variable,

$$D = G + \mathrm{Tr} + (1+r)B - \tau_k r \int_{a \geq 0} a \, d\theta - \tau_w w N , \tag{8}$$

where B and G are current government debt and purchases, respectively. We assume that D is to be financed through lump-sum taxes, Γ, and newly issued debt. It follows that:

$$D = \Gamma + B' . \tag{9}$$

There is a fiscal rule whereby lump-sum taxes are imposed based on the difference between the steady-state level of the gap to finance, $\bar{D}$, and its current level, D, so that when this difference is zero lump-sum taxes remain at their steady-state level, $\bar{\Gamma}$. Formally,

$$\Gamma - \bar{\Gamma} = \phi(D - \bar{D}) \,. \tag{10}$$

If ϕ is one, then all the gap is financed through lump-sum taxes. If ϕ is close to zero but large enough so as to ensure stability of the debt level, then the gap is largely financed through issuing debt, with taxation being postponed into the future. The second case is of great interest to us; our simulations will therefore be conditioned on very low levels of ϕ.[6]

2.3 Equilibrium

The steady-state equilibrium in this economy is standard. Given a transition matrix Π for idiosyncratic productivity, a set of government policies $(\tau_k, \tau_w, \text{Tr}, G)$, a fiscal rule summarized by ϕ, and assuming that any deviation to default is not coordinated among households, we define a *recursive competitive equilibrium* as a belief system H, a pair of prices (r, w), a measure defined over the set of possible states θ, paths for government debt, lump-sum tax and gap to finance (B, Γ, D), a pair of value functions $\upsilon(x, \theta)$ and $\underline{\upsilon}(z, \theta)$, and individual policy functions $(a', c, n) = (a(x, \theta), c(x, \theta), n(x, \theta))$, such that:

1. Each agent solves the optimization problem (1);
2. Firms maximize profits according to (6) and (7);
3. The government balances its budget according to (8) and (9);
4. All markets clear:

$$K' + B' = \int a(x, \theta) \, d\theta \tag{11}$$

$$N = \int n(x, \theta) \, z \, d\theta \tag{12}$$

$$\int c(x, \theta) \, d\theta + K' + G = (1 - \delta)K + AK^{\alpha} N^{1-\alpha} \,; \tag{13}$$

5. The belief system H is consistent with the aggregate law of motion implied by the individual policy functions;
6. The measure θ is constant over time.

The definition of an equilibrium with a transition follows naturally from the previous one although at the cost of a heavier notation, so we economize on space and omit it. Briefly, as will be stressed in Section 4 our transition is triggered by the unexpected introduction of a perfectly credible and deterministic change in the trajectory of either government transfers

[6] We use only lump-sum taxation in the fiscal rule so as to avoid changes in the tax rates interacting with the channel under scrutiny. For an evaluation of the relation between the tax rates and the endogenous borrowing constraints, see Ábrahám and Cárceles-Poveda (2010).

or purchases, along with a fiscal rule. We assume that in the transition agents can perfectly foresee the evolution of aggregate variables, including the borrowing limits, thus making sure that off-equilibrium paths are not observed. Aggregate uncertainty is therefore not considered in this analysis.

3 Steady-state calibration

In this section, we discuss the steady-state calibration of the model. Full details about the computational procedure are given in Section A of the appendix. We calibrate the model at quarterly frequency and present the relevant calibration targets in Table 1.

The idiosyncratic productivity is modeled in most of the literature by a persistent AR(1) process, as in Krueger and Perri (2005), sometimes coupled with a white noise component, as in Storesletten et al. (2004). In these papers labor supply is exogenous. Hence, we borrow the time-varying component of labor productivity from Floden and Lindé (2001), who use endogenous labor supply,

$$\log(z_t) = \rho \log(z_{t-1}) + \eta_t \, ,$$

where ρ defines the persistence of the process and η_t is a serially uncorrelated and normally distributed perturbation with variance σ^2_η. The parameters ρ and σ^2_η are set so as to match the yearly autocorrelation and variance of the labor productivity process, which are 0.9136 and 0.0426, respectively (as estimated by Floden and Lindé 2001). In order to discretize the productivity process, we use the Rouwenhorst method (see Kopecky and Suen 2010) with 7 levels of productivity.

The implied transition probability matrix, Π, is characterized by non-zero entries everywhere. This implies that, whatever the productivity of the household in a given period, it can have the lowest productivity in the next period (possibly with a very small probability). Therefore, the only borrowing limit such that all households will be able to pay back their debt irrespective of the productivity shock that hits them is the tightest among the limits defined in (5); this implies that the borrowing limit is parameterized in the lowest productivity level, $z_1 \in \mathcal{Z}$. We define this borrowing limit to be $\underline{a}(\theta') = \underline{\hat{a}}_{z_1}(\theta')$.[7]

Figure 1 shows a graphical representation of this constraint together with a set of value functions. Specifically, the figure depicts pairs of value functions associated with the three lowest levels of z, as formalized in (4), as a function of assets. According to equation (3), the autarky value functions are flat, whereas the equilibrium value functions have a positive slope. The borrowing constraint is found at the intersection between the equilibrium and the autarky value functions parameterized by the lowest level of productivity, z_1; this is represented by the vertical line in the figure. The reaction of these value functions to the fiscal policies will determine the new position of the borrowing constraint.

[7]Having zero-valued elements in Π would imply the presence of several borrowing constraints contingent on, at least, some levels of productivity. Working with more borrowing constraints would make, on the one hand, the quantitative analysis more realistic but, on the other hand, the computational procedure more cumbersome without adding relevant theoretical insights.

Quantitatively, the resulting borrowing limit is such that a household with the average income can borrow up to roughly 45% of its yearly income.[8] The steady-value value of the borrowing limit is obtained indirectly, by matching the actual credit-to-output ratio. Since in our model there is only unsecured credit, we calibrate the model's credit using data for total revolving credit.[9] As in Antunes and Cavalcanti (2013), we target a credit-to-output ratio of 8%, which is the average pre-crisis period, fixing γ at 0.956.

Given the targeted credit-to-output ratio, our economy is characterized by roughly 9% of agents at the borrowing constraint (labeled as constrained agents) and a total of roughly 24% of borrowers, which are values close to the actual ones in the U.S. economy (Ábrahám and Cárceles-Poveda 2010, Jappelli 1990).[10] Constrained agents are a sub set of borrowers. Unconstrained agents represent about 91% of the population: 76% of the population hold a positive level of assets, while 15% of the population hold negative assets, that is, are borrowers.

Figure 1 provides an interesting insight about the type of household that would be more tempted to default, which is in fact in line with the empirical evidence. It can be seen that, as the household's productivity increases, both types of value functions move up; however, the equilibrium value function moves up by more than the autarky value function. Hence, the temptation to declare default, that is, to choose autarky relative to honoring debt commitments, decreases with the household's productivity, for any level of asset holdings. This is due to the incompleteness of the market: *ceteris paribus*, a high income household would loose more by defaulting than a low income household, since the opportunity cost of a permanent preclusion from self-insurance is higher for the former.

Using the standard natural borrowing limit instead of our participation constraint would yield values for the credit-to-output ratio and the percentage of households at the constraint considerably far from their actual values. The first measure would be around 30% and the percentage of constrained households would be virtually nil. This is due to the way the natural limit is implemented; it is computed so that households would consume zero at the constraint, conditional on a long string of realizations of the worst productivity shock.

Table 2 compares the model wealth distribution with that of the U.S. as reported by Castañeda et al. (2003). The profile of the wealth distribution in our model does a reasonable job at mimicking the U.S. wealth distribution. In particular, as in the data, households in the first quintile hold negative wealth and those in the fifth quintile hold most of the existing wealth. However, as usual in this type of model and with the specific assumptions about the stochastic behavior of the idiosyncratic shocks, the model does not generate enough inequality, especially in the upper tail of the asset distribution.

Notice that in the stationary equilibrium the government budget is balanced and public debt

[8] If we wanted to express the borrowing limit in percentage terms of the quarterly income, the number would become roughly 190%, which is the value reported in the x-axis of Figure 1.

[9] Credit in the model is the aggregated amount of negative net wealth.

[10] As explained in Section A of the appendix, we use a grid for the asset holdings throughout our computations. We define agents to be constrained if they seat in the grid points which are in a eye-ball of 5% around the borrowing limit. Equivalently, constrained agents are defined to be the ones seating in the two grid points nearest to the borrowing limit.

Table 1: Steady-state calibration.

Parameter	Value	Observation/Target
A	1	Normalization
α	0.36	Share of capital in production
δ	0.025	Capital-to-output ratio of 2.6 (yearly)
σ	2	Standard in the literature
ψ	0.67	Frisch elasticity of 1.5
β	0.9894	Real interest rate of 1%
ρ	0.977	Floden and Lindé (2001)
σ_η	0.11	Floden and Lindé (2001)
χ	0.4	Average labor supply normalized to 1
τ_w	0.27	Domeij and Heathcote (2004)
τ_k	0.4	Domeij and Heathcote (2004)
γ	0.956	Credit-to-output ratio is 8% (yearly)

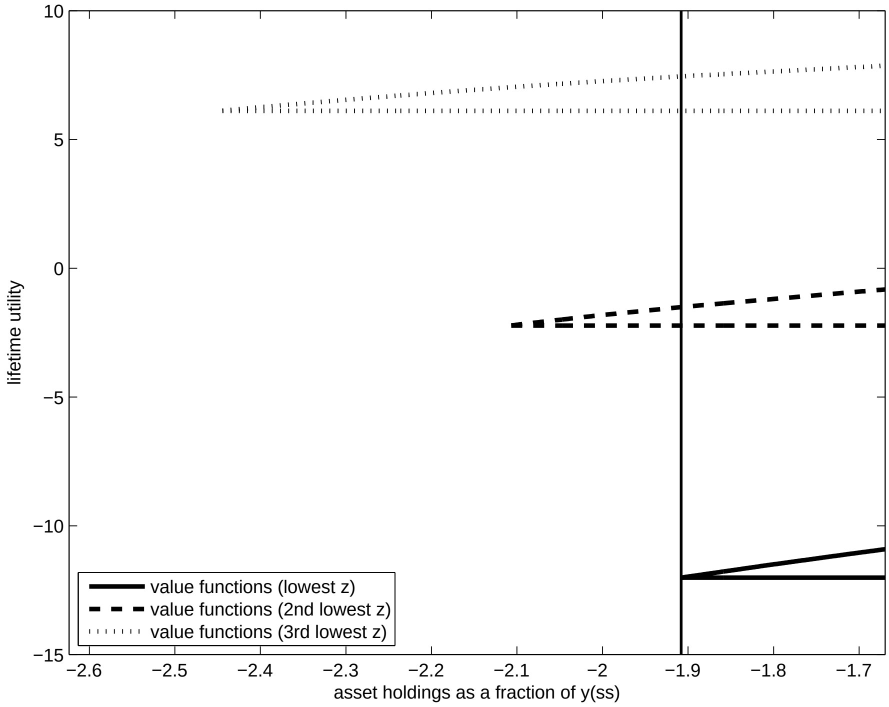

Figure 1: Selected pairs of value functions and the borrowing limit in the steady state. The flat lines correspond to the value functions in autarky, $\underline{v}(z,\theta)$, for different levels of productivity. The lines with positive slope refer to the equilibrium value functions, $v(x,\theta)$, for different levels of productivity. The relevant borrowing limit, $\underline{a}(\theta)$, is identified by the vertical line. y(ss) stands for steady-state output.

Table 2: Distribution of wealth: U.S. economy (Castañeda et al. 2003) vis-à-vis the model. All values are in percentage, except the Gini index which is expressed in natural units.

	Gini index	Quintiles				
		First	Second	Third	Fourth	Fifth
Data	0.78	-0.39	1.7	5.7	13.4	79.5
Model	0.67	-3.0	1.7	10.9	26.5	63.9

is assumed to be zero; hence, the gap to finance, D, is zero as well. Moreover, $\text{Tr} = \Gamma = 0$. We also perform simulations starting from positive and large levels of public debt (see robustness exercises mentioned in Sections 4.1 and 4.2). Given the chosen values for the tax rates, we identify a government consumption of around 21% of steady-state output, which is close to the actual measure in the U.S. data.

Section B of the appendix depicts the agents' policy functions for consumption and labor, for different productivity levels.

4 Transition with public debt expansions

We perform two sets sets of exercises. In the first, a public debt expansion is used to transfer resources to households in a lump-sum fashion. In the second, the public debt expansion is used to increase government purchases. The policies are unexpected by the households. The results of the first exercise are presented in this section, while the results associated to the increase in government purchases are presented and briefly commented upon in Section D of the appendix.

Technically, we set the simulation horizon to 600 quarters. We then iterate on the path of prices, the set of time-dependent policy functions and the time-dependent wealth and productivity joint distributions, under the assumption of perfect foresight, until we have a fixed point in these objects. Section A in the appendix gives a detailed account of the computation of the transition.

Below we report and explain the effects of the debt expansion on the borrowing constraint and on other variables. We also describe the consequences of the constraint's movement on the households' reaction. Finally, we show the implications on welfare of considering the borrowing limit as an endogenous variable.

4.1 The dynamics of the borrowing limit, interest rate and credit

The transfers policy We simulate a debt expansion that finances an increase in transfers, denoted Tr, that is uniform across agents. On impact, which we define to occur at $t = 1$, transfers increase by 1% of steady-state output and then decay following an AR(1) with persistence 0.95, as estimated by Leeper et al. (2010). In order to postpone lump-sum taxation, denoted Γ, into the future, we set the parameter ϕ in the fiscal rule (10) at a low level, 0.02, which is still large enough so as to ensure stability of the debt level. As a result, the debt-to-output ratio increases up to a maximum of 12% around the 30^{th} quarter (3% in yearly terms) and then slowly comes back to its original steady-state level. The evolution of the fiscal variables are depicted in the two top panels of Figure 2.

The credit tightening process Figure 2 presents selected reactions to the above-described transfers policy. In particular, solid lines are generated within our "baseline model", where the borrowing limit is allowed to evolve endogenously. Dashed lines, instead, are drawn conditional on keeping the borrowing limit fixed throughout the transition, implying that the relevant

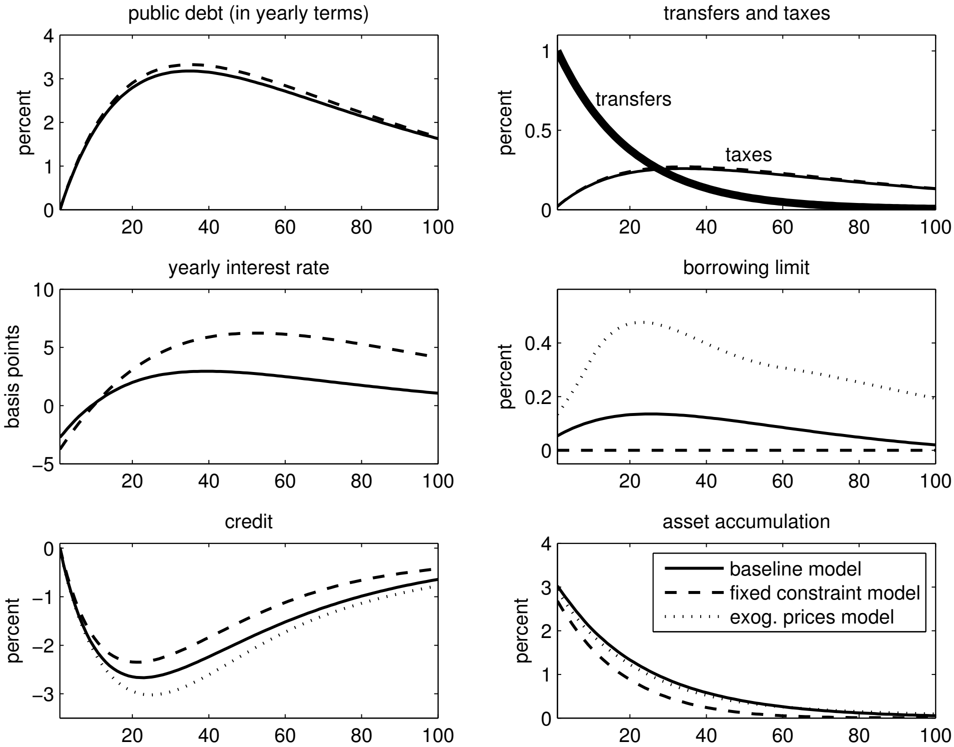

Figure 2: Selected reactions to the transfers policy. All variables are expressed in deviation from steady-state levels. The deviations for public debt, transfers, taxes and the borrowing limit are normalized by steady-state output. Positive deviations for the borrowing limit mean a tightening. Asset accumulation is the difference between the asset level in $t+1$ and the steady-state level, normalized by steady-state investment. The x-axes are in quarters.

borrowing constraint becomes $a' \geq \underline{a}$, where $\underline{a}$ is the endogenously determined borrowing limit using the baseline model in the steady state. The latter specification is labeled as "fixed constraint model". The gap between the solid and the dashed lines represents the part of the reactions attributable to the dynamics of the borrowing limit.

Over time, issuing public debt positively impacts on the interest rate.[11] All else equal, this makes the option of staying in the market relatively worse than going to autarky, giving the borrowers a higher incentive to declare bankruptcy. Knowing that, lenders are willing to lend less which endogenously tightens the household borrowing constraint, meaning that the maximum amount that can be borrowed is reduced.[12] Such tightening is persistent over time; ten years after the beginning of the debt expansion, the borrowing limit is still not back to its steady state. On the one hand, the tightening forces constrained agents to deleverage. On the other hand, unconstrained agents realize that, all else equal, their asset position will be closer to the borrowing limit; hence, their precautionary saving motive is reinforced. The unconstrained

[11]Notice two things here. First, the interest rate decreases on impact, but after roughly two years it is already higher than its steady-state level, and reaches its peak in ten years. After that, it slowly comes back to the steady-state level. The initial fall in the interest rate is explained by the fall on impact of labor, whose details are described in Section 4.2. Second, the elasticity of the interest rate to public debt obtained in our case is of the same order of magnitude of the elasticity obtained in Aiyagari and McGrattan (1998), despite the circumstance that they perform a steady-state analysis.

[12]The movement of the equilibrium and autarky value functions—which generate the tightening—is carefully shown and explained in Section C.1 of the appendix. This section also shows the dynamics of the policy functions in the moment of the implementation of the fiscal policy.

savers will save more, while the unconstrained borrowers will cut on their debt.[13] As a result, the fall (increase) in credit (assets) is larger in the baseline model than in the fixed constraint model.[14]

Quantitatively, within the baseline model the borrowing limit tightens by an average of 0.12% of steady-state output during the first five years of the debt expansion. Furthermore, considering the same five-year horizon, a tightening in the borrowing limit of 1 unit of steady-state output is associated with an average fall in credit of 0.49 units of steady-state output.[15]

As explained above, the tightening positively influences the households' desire to deleverage or increase asset holdings. This generates a downward pressure on the interest rate, which is indeed lower in the baseline model than in the fixed constraint model.[16] Let us decompose the effects generated by the tightening. In particular, we want to isolate the effects due to the movement of the borrowing limit alone from those generated by the change in prices. For this purpose, we simulate an off-equilibrium version of the model where the borrowing limit is allowed to react to the fiscal policy but prices are unchanged from the fixed constraint economy, meaning that prices do not in turn react to the shift of the borrowing limit. This economy is labeled as "exogenous prices model" and is graphically described by the dotted lines in Figure 2.[17] The reaction of the borrowing limit in the exogenous prices economy is much more pronounced than that observed in the baseline model, which means that in the baseline model the interest rate fall generated by the tightening mitigates the tightening itself. For instance, five years after the policy implementation, the interest rate differential contributes to dampen the tightening obtained in the exogenous prices economy by roughly 70%. Such gap in the interest rate contributes also to mitigate the reaction in terms of credit.

Robustness exercises In Section C.2 of the appendix we present a number of robustness exercises targeted to the dynamics of the borrowing constraint. First, we simulate a version of the model in which prices (interest and wage rates) are kept fixed at their steady-state level; this simulation shows that the dynamics of the interest rate generated by the fiscal policy is crucial for the occurrence of the tightening.[18] Second, we perform our simulations within a model that uses an alternative borrowing limit, namely an endogenous ad hoc limit. Third, we simulate the economy using different steady-state levels of public debt. Fourth, we produce additional robustness evidence by changing the parameter of the fiscal rule, ϕ, and the way public debt expansions are financed. These robustness exercises—with the exception of the

[13]The detailed reactions of both constrained and unconstrained households are reported in Section 4.2.

[14]Since we work with net wealth, deleveraging or, equivalently, cutting debt, contributes to a positive asset accumulation.

[15]The fall in credit is computed as the difference between credit in the baseline model and credit in the fixed constraint economy. In this case this difference is negative in all quarters.

[16]In fact, at the very beginning of the debt expansion, the interest rate values are very similar in the two economies. This is partly explained by the behavior of labor in the two economies; see Section 4.2.

[17]More specifically, the exogenous prices case is the simulation such that the borrowing constraint is allowed to change endogenously but the paths of prices, debt and taxes are kept at the levels of the fixed constraint case.

[18]This simulation is different from the exogenous prices one; in the latter prices are those of the fixed constraint economy, whereas in the former prices remain at their steady-state level.

first—deliver results which are similar to those obtained using our baseline specification.

4.2 Dynamics at the individual and aggregate level

In this section, we first present the reaction to the transfers policy of two groups of households: the constrained and the unconstrained. We focus on how the movements of the borrowing limit influence differently the reaction of these two groups of agents to the fiscal policy. Second, we aggregate up the individual reactions and see how and by how much the dynamics of the borrowing limit affects the responses of several aggregate variables, such as labor, consumption, capital and output. Third, we show the importance of the reaction of unconstrained households in explaining the effects generated by the dynamics of the borrowing limit.

Heterogeneous reactions Figure 3 shows the average reaction in assets, labor and consumption for both constrained (left column) and unconstrained households (right column).[19]

Let us focus on the reaction of the households under the fixed constraint model, which is represented by the dashed lines. Constrained households use the transfers received from the government to increase their consumption given that their marginal propensity to consume is the highest in the economy. Over time, they start decreasing consumption because of the higher future taxation. The dynamics of labor effort roughly mirrors that of consumption. Regarding asset accumulation, households deleverage because they now rely more on the received transfers than on the (more expensive) borrowing for targeting the desired level of consumption.

In terms of consumption and labor, unconstrained households react to the transfers policy much less than the constrained. As a matter of fact, the unconstrained use a considerable part of the received transfers to buy assets (if they are lenders), or to decrease their indebtedness (if they are borrowers), in order to be able to pay the higher future taxes while keeping a smooth profile for consumption. The rise in saving occurs also because of the increase over time of the return on assets: as highlighted by Aiyagari and McGrattan (1998), this helps savers self-insure so that their asset position will happen to be farther away from the borrowing constraint.[20]

What does the tightening of the borrowing constraint add to these reactions? Let us answer to this question in two steps. First, we analyze the consequences of the tightening depurated from its price effects by comparing the reactions of the exogenous prices model (dotted lines) with those of the fixed constraint model (dashed lines). As stated in Section 4.1, because of the tightening constrained households must deleverage, which implies that their asset accumulation is stronger in the exogenous prices case than in the fixed constraint case. This extra deleveraging is obtained by cutting consumption and supplying more labor. Unconstrained agents behave

[19] The reaction of labor in the first period of the transition for constrained households is calculated as $\int_{a \in \mathcal{V}(\underline{a})} \left(\frac{n_1(x,\theta) - n(x,\theta)}{n(x,\theta)} \right) d\theta$, where $n_1(x, \theta)$ is the labor policy function in the first period of the transition, $n(x, \theta)$ is the policy function in the steady state and $\mathcal{V}(\underline{a})$ is a tight neighborhood of $\underline{a}$ as defined in footnote 10, Section 3. The reactions in the following periods are calculated in the same fashion. The computation is similar for the unconstrained with the appropriate change in the integration domain. Finally, the same logic is used to compute the heterogeneous responses for consumption and asset holdings.

[20] Notice that the unconstrained borrowers will decrease their indebtedness also because of higher borrowing costs.

similarly but with lower intensity: they do not have to adjust forcefully and expand their asset holdings to hedge against the increased risk of becoming constrained in the future.

In the second step, we analyze the price effects generated by the tightening, which are illustrated by the comparison between the reactions of the baseline model (solid lines) with the ones of the exogenous prices model (dotted lines). It is worth noting that relative lower (higher) interest (wage) rates exert substitution and income effects which can have contrasting impacts on the households' reactions. A particular effect seems to dominate in the case of constrained households: the lower interest rate dampens the tightening itself, so that this type of households needs to deleverage much less in the baseline model than in the exogenous prices model. In contrast, for unconstrained agents these price effects mildly amplify the reactions obtained within the exogenous prices economy. In brief, under the transfers policy the price effects dampen the reactions of constrained agents, while amplifying those of the unconstrained. Interestingly, under the purchases policy the price effects dampen the reactions of both categories of households; see Section D.3 in the appendix.

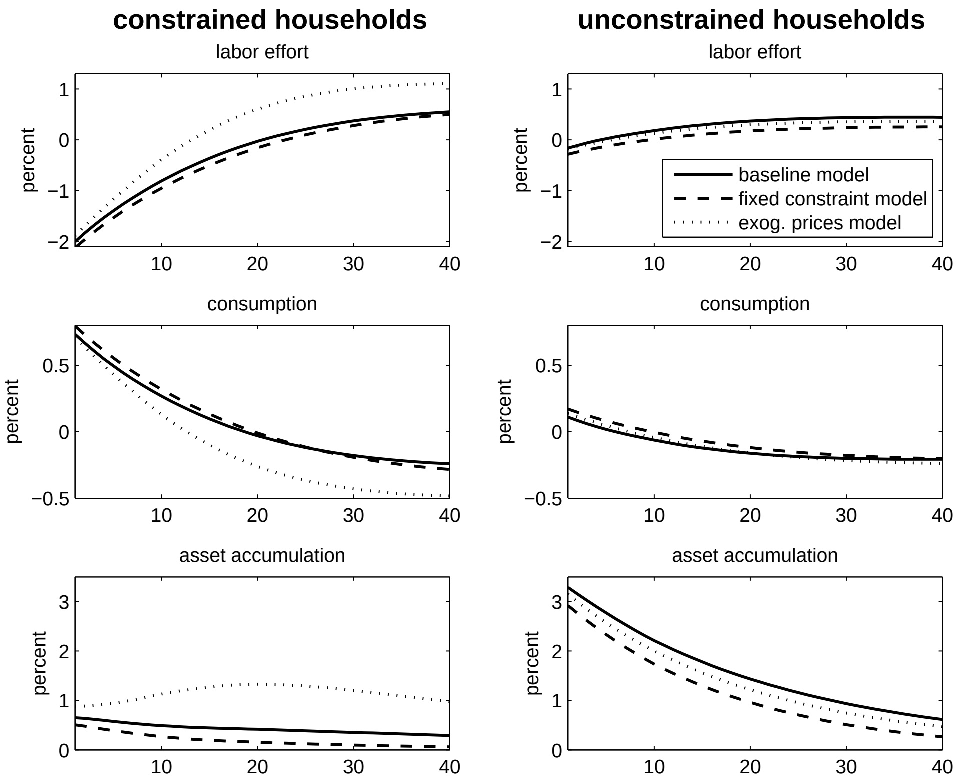

Figure 3: Heterogeneous effects of the transfers policy. The average reactions of constrained agents are on the left column. The average reactions of unconstrained agents are on the right column. All variables are expressed in deviation from steady-state levels. Asset accumulation is the difference between the asset level in $t+1$ and the steady-state level, normalized by steady-state investment. The x-axes are in quarters.

Aggregate reactions What is the aggregate impact of the government debt expansion on the economy? How much of such an impact is explained by the tightening of the borrowing constraint? An upfront answer can be given by looking at the (cumulative) output multiplier in

Figure 4.[21] The fixed constraint model generates a multiplier close to -0.2 on impact and -0.45 over a five-year horizon. Considering the movement of the household borrowing constraint and its price effects significantly dampens the fall in output: for example, the multiplier becomes roughly -0.25 in the baseline model over a horizon of five years. The price effects generated by the tightening explain a sizable part of the output fall; see the dynamics of the dotted lines.

In this framework, the tightening produces an increase in output because of both the labor and the capital dynamics. First, the tightening, on average, induces households to exert a stronger labor effort.[22] Second, regarding physical capital, the higher interest rate crowds it out; however, capital is crowded out less in the baseline model than in the the fixed constraint model because of the lower interest rate in the former economy. Notice further that, consistently with the heterogeneous reactions described above, the tightening, on average, depresses consumption. The wage rate is higher in the baseline economy at almost any horizon.

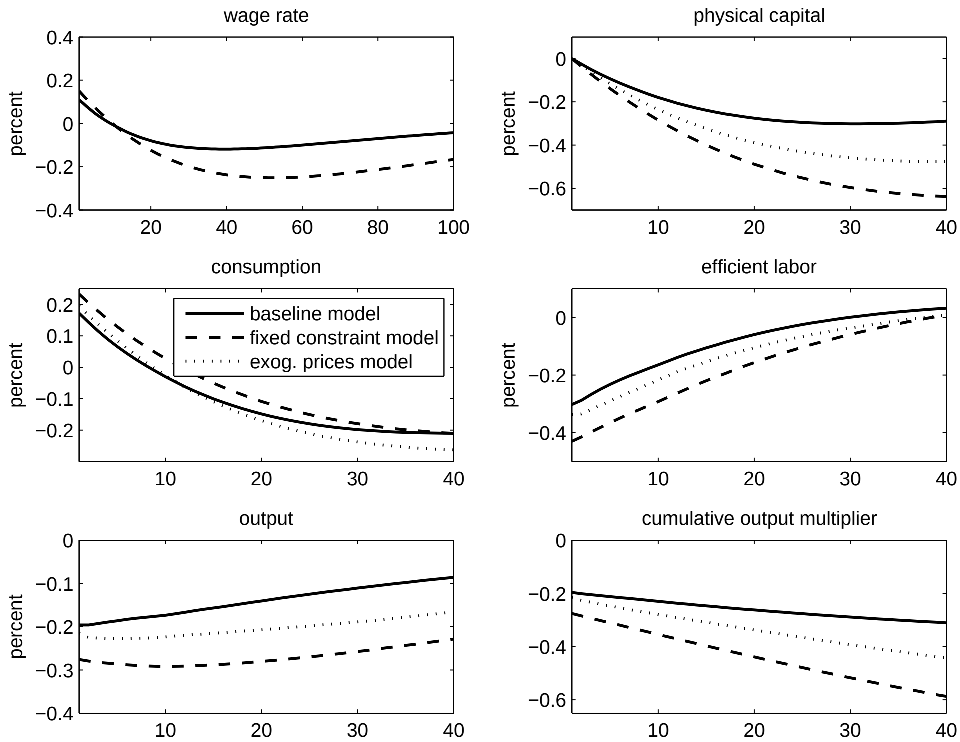

Figure 4: Aggregate effects of the transfers policy. All variables are expressed in deviation from steady-state levels. The cumulative output multiplier is calculated following Uhlig (2010); see footnote 21. The x-axes are in quarters.

Why the reaction of unconstrained households matters Does the magnitude of the effects generated by the tightening significantly depend on the calibrated share of constrained

[21] The output multiplier is a good indicator to evaluate the aggregate impact of a given policy because it relates the effects of the policy with the magnitude of the policy itself. Following Uhlig (2010), the output multiplier t quarters after the policy implementation is calculated as $\sum_{k=0}^{t}(1+r_{ss})^{-k}\hat{Y}_k/\sum_{k=0}^{t}(1+r_{ss})^{-k}\hat{\text{Tr}}_k$, where $\hat{Y}_k$ and $\hat{\text{Tr}}_k$ represent the actual deviations of output and transfers from their steady states, respectively, with r_{ss} being the steady-state real interest rate.

[22] Within otherwise standard incomplete-markets models, the sign of the labor reaction to a tightening can be negative. For example, Kehoe et al. (2016) show that such a sign depends on the form of the household's utility function. Further, Huo and Ríos-Rull (2015) show that considering search frictions in some consumption markets suffices to generate a drop in employment as a result of tighter credit conditions.

agents? Overall, the answer is negative. Table 3 reports the effects of the tightening (depurated from its price effects) on the reactions to the fiscal policy of different groups of households: the constrained, the unconstrained, and the wealthiest 5%. The table also reports the contribution of the reactions of each group of households to the aggregate, or total, reactions.[23]

As already stressed, constrained agents react the most to the movement in the borrowing constraint. In particular, their average reactions are roughly five times larger than those of the unconstrained in terms of labor and consumption. In terms of asset accumulation, this proportion is about three times and a half larger. However, the sum of the reactions of unconstrained households contributes by almost three fourths to the aggregate reactions. Hence, setting the percentage of constrained agents in the steady state to values lower than 9% would only mildly influence the magnitude of the aggregate effects due to the shift of the borrowing limit.

Table 3: Average changes of policy functions and contributions due to the movement of the borrowing constraint. The differences between the average reactions of the exogenous prices model and those of the fixed constraint model ("Diff.") for several groups of households are reported. The contributions of the reactions of each group of households to the aggregate reactions ("Contrib.") are also reported. All the figures represent an average over the first five years of the policy. All values are in percentage.

	Constrained		Unconstrained		Wealthiest 5%		Aggregate	
	Diff.	Contrib.	Diff.	Contrib.	Diff.	Contrib.	Diff.	Contrib.
Labor effort	0.58	33	0.11	67	0.08	2.5	0.16	100
Consumption	-0.19	34	-0.04	66	-0.027	2.5	-0.05	100
Asset accumulation	0.89	26	0.25	74	0.18	2.9	0.31	100

Interestingly, the average reactions of the wealthiest households—a sub set of unconstrained agents—to the movement in the borrowing limit are relatively small because they are very far from such a limit. However, these reactions are not nil because, even for the richest households, the probability of becoming debt-constraint in the future increases after the debt expansion, factoring out the price effects generated by the tightening.

Robustness exercises We perform the same type of robustness exercises described in Section 4.1; these exercises are however targeted to the magnitude of the aggregate effects generated by the tightening. These exercises deliver results which are similar to those obtained using our baseline specification. Section C.3 of the appendix details these results.

Finally, we produce an additional exercise: a "crisis experiment". We want to see how public debt expansions influence an economy in which households' financial conditions tighten—featuring a stricter borrowing limit and falling credit—and a recession is in place. Both the transfers and the government purchases policies contribute to a more marked fall in credit and a further tightening in the limit, whilst they do not produce significant effects on output. Section E of the appendix presents detailed results.

[23]The contribution of labor of constrained agents is the average reaction of these households multiplied by their respective population weight, for any quarter. These measures are then averaged out over a five-year horizon. The same logic applies to the other groups of households. The calculations for both consumption and asset accumulation are performed in the same fashion. Notice that summing up the contributions of unconstrained and constrained households, for a certain model variable, yields a total of 100 percent.

4.3 Welfare implications

This section studies the effects of the debt-financed transfers policy on the households' welfare. In particular, we focus on how the welfare analysis changes whether or not we consider the endogenous borrowing constraint in the model. Figure 5 presents the welfare gains along the household wealth distribution for the baseline (solid line) and the fixed constraint (dashed line) models.[24]

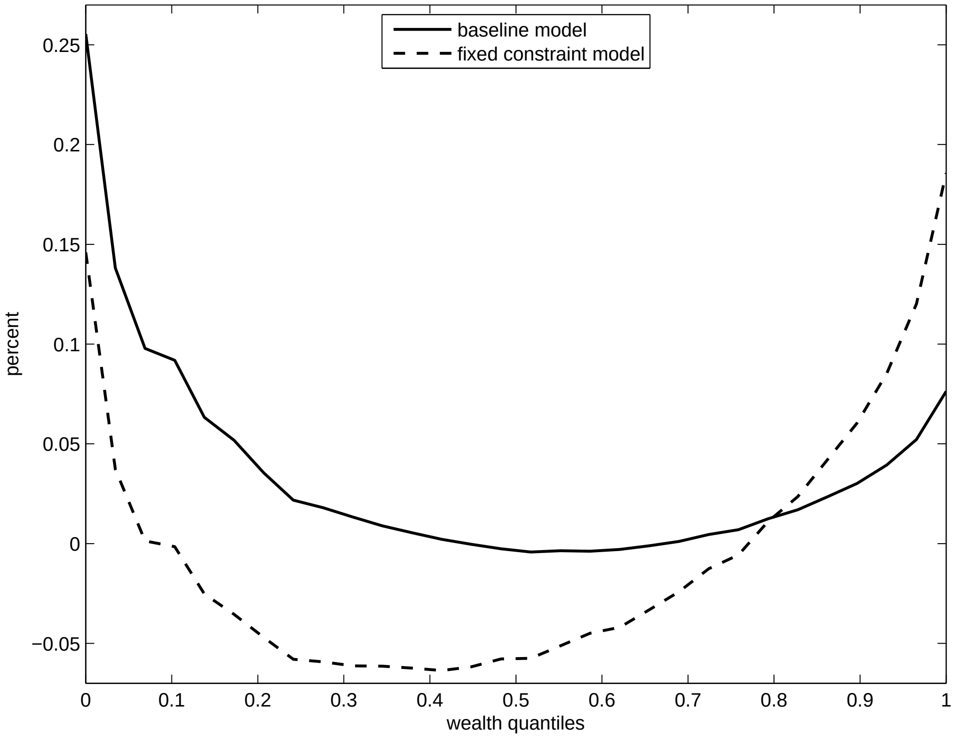

Figure 5: Welfare effects of the transfers policy by wealth quantiles. Positive values represent a welfare gain measured as the consumption-equivalent change relative to the initial steady state. See the definition in footnote 24.

Let us focus on the welfare effects of the policy within the fixed constraint economy. The policy produces positive effects for the households in the bottom 10% of the wealth distribution, a group which corresponds roughly to constrained households. These agents use the transfer to consume more and rely less on borrowing, which is costlier after the policy implementation. Other forces, like a higher future taxation, are not enough to nullify the mentioned welfare increase. The rest of the borrowers together with the households up to roughly the fourth quintile of the distribution suffer a welfare loss. This loss can be due to higher borrowing costs, lower wages and higher future taxation. For households in the top quintile of the distribution, the policy generates a welfare gain. A plausible cause is the increase in the remuneration of the

[24] To assess the welfare gain of a policy relative to a certain *status quo* we proceed as follows. For each point (a, z) of the state grid, we compute the consumption-equivalent variation as the constant percentage change in consumption along the transition path such that, at the moment of the implementation of the policy, the agent is indifferent between staying in the *status quo* or switching to the economy under the policy. The welfare gain for a specific set of agents—such as for example agents with the same level of wealth, or agents within a given wealth range, or all agents—is computed by averaging out the consumption-equivalent variation over that set, using the measure of agents as weights.

asset holdings, which represent the main source of their income. Table 4 reports the political support and the average welfare gain associated to the policy.[25] The majority of the households does not support the implementation of the transfer policy: only around 30% of the population is in favor.

Table 4: Political support to, and average welfare gain of, the transfers policy relative to the *status quo* (no policy implementation). Political support is the fraction of agents for whom the welfare gain of the policy change is positive. The average welfare gain is the average across all states of the state specific welfare gain, using the measure of agents as weight. All values are in percentage.

	Political support	Avg. welfare gain		
		Constrained	Unconstrained	Total
Fixed constraint model	33	0.0615	-0.0125	-0.0051
Baseline model	77	0.1638	0.0206	0.0349

How does considering the dynamics of the borrowing constraint influence the welfare results? All in all, the welfare associated to the baseline model is higher relative to the fixed constraint model for all households up to roughly the fourth quintile. For richer households, the opposite holds. The tightening can produce different effects on welfare. In principle, the tightening forces constrained households to borrow less and, at the same time, limits the maximum quantity of borrowing for anyone else. This fact should produce welfare losses for all the households. However, we have seen that the tightening generates also price effects: relatively higher (lower) wage (interest) rates. These effects represent a welfare gain for the borrowers (who can borrow cheaper) and the middle-class households (who mostly rely on their labor income) and a welfare cost for the rich (who mostly rely on their asset income). These price effects seem to have a larger impact on households' welfare vis-à-vis those caused by the pure movement of the borrowing constraint. As a result, by looking at Table 4, the majority of households (almost 80%) would vote in favor of the implementation of the policy in the baseline economy. In brief, under this specific transfers policy allowing for an endogenous borrowing constraint significantly alters the welfare results.

At first glance, our results on the effect of the tightening on welfare seem surprising. In fact, they gather support from other findings in the literature. For example, Dávila et al. (2012) show that the competitive equilibrium of a heterogenous agent model with incomplete financial markets is constrained inefficient because of the presence of a pecuniary externality. That is, agents do not take into account that their own actions influence prices. Hence, if a social planner can choose a policy function for each agent taking price effects into account, then it can improve the allocation. They show that, for the standard Aiyagari (1994) model, the planner wants agents to save more in order to increase (decrease) the wage (interest) rate. Wealth-rich agents suffer a welfare loss but the wealth-poor agents are better off. Since the effects are positive for agents with higher marginal utility of consumption, social welfare increases. Farinha Luz and Werquin (2011) obtain a similar result performing a different analysis. They show that

[25] Political support is the fraction of agents for whom the welfare gain of the policy is positive. The average welfare gain is computed as explained in footnote 24.

the Huggett (1993) economy, which does not consider capital accumulation, can be constrained inefficient as well. Hence, imposing a stricter borrowing limit can be welfare improving because it forces wealth-poor agents to save or deleverage more, depressing the interest rate.

The relation between the households' welfare and the sign of the output multiplier is worth noting. Under the baseline model, the transfers policy delivers negative output multipliers with a majority of households supporting it. This suggests that the sign of fiscal multipliers and of the households' welfare are not necessarily positively correlated as discussed in Kolosova (2013) using a heterogenous agents model with ad hoc borrowing limits, and Bilbiie et al. (2014) using a standard New Keynesian representative agent model.

5 Conclusions

This work shows that considering the endogeneity of the household borrowing constraint is important for carefully evaluating the effects of archetypal fiscal policies entailing government debt expansions. To account for this, we use an incomplete-markets model featuring uncollateralized credit and borrowing constraints that allow for limited commitment in the repayment obligation of the borrower.

Issuing public debt creates a tension in the households' financial conditions. On the one hand, it can help relax the household borrowing constraint through a rise in the return on assets (Aiyagari and McGrattan 1998) or a change of the collateral value (Angeletos et al. 2016). On the other hand, it can tighten the constraint due to an increase in the borrowing costs. The literature has typically emphasized the first set of channels. We instead focus on the second channel.

The tightening generated by the debt expansions fosters a deleveraging process in terms of private credit and an increase in (precautionary) saving that, in turn, produce a downward pressure on the interest rate. This partly mitigates the tightening itself. More specifically, the tightening exerts effects on the economy through two channels: first, the reduction of the maximum quantity that households can borrow; second, the price effects induced by the movement in the borrowing limit along the whole transition path. Both channels account for a sizeable part of the households' reaction to the fiscal policies. As typical in the class of incomplete-markets models used in the present analysis, the magnitude of the households' reaction depends on the respective level of wealth. The welfare implications of the policies are influenced by the consideration of the endogeneity of the constraint as well.

We see two possible avenues for future research. First, our results are obtained using a model which considers only unsecured consumer credit. As we conjectured in the Introduction, our channel survives even if we consider a borrowing constraint characterized by collateralized credit. However, it would be interesting to study how, and by how much, the inclusion of collateral would change the present results. In this vein, Angeletos et al. (2016) provide some interesting insights within a model in which the households' wealth distribution is not a state variable of the problem. Second, government debt expansions generate movements in the household borrowing constraint that typically trigger heterogenous reactions among constrained and wealth-rich households. This fact can have implications for the level of the wealth inequality in the economy that merit further inquiry.

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A Computational procedure

The objective of this section is to explain in detail the computational procedure for calculating both the stationary distribution and the transition of the model economy after a policy or parameter change.

A.1 The solution method

As explained in Section 3, the relevant borrowing limit in the model economy, $\underline{a}(\theta')$, is unique and parameterized in the lowest z.[26] The problem to solve is therefore:

$$v(a,z,\theta) = \max_{c,n,a'} u(c,n) + \beta \mathbf{E}\left[v(a',z',\theta')|z\right] \tag{14}$$

subject to

$$c + a' = (1 + r(1-\tau_k \mathcal{I}_{a\geq 0}))a + wnz(1-\tau_w) + \mathrm{Tr} - \Gamma \tag{15}$$

$$a' \geq \underline{a}(\theta')\,. \tag{16}$$

[26]Despite we work with a unique borrowing limit, we should mention that the proposed solution method is robust to the consideration of borrowing limits contingent on different z's.

We use a direct solution method for solving problem (14). The problem is a mixed-constrained optimal control problem because of the coexistence of equality and inequality constraints. We will economize on notation and drop explicit reference to measure θ. Consider the Lagrangian function:

$$\begin{aligned}\mathcal{L}(c,n,a') = {} & u(c,n) + \beta\,\mathrm{E}\left[v(a',z')|z\right] + \\ & \left((1+r(1-\tau_k\mathcal{I}_{a\geq 0}))a + wnz(1-\tau_w) + \mathrm{Tr} - \Gamma - c - a'\right)\lambda + (\underline{a}-a')\,\mu + (\overline{a}-a')\,\gamma\end{aligned} \tag{17}$$

where we assume that there is an absolute upper level for asset holdings, $\overline{a}$. This will have to be confirmed once we solve the problem numerically. In practice it suffices to set it to a sufficiently high positive value. The necessary conditions for an optimum of the above problem are:

$$u_c(c,n) - \lambda = 0 \tag{18}$$

$$u_n(c,n) + \lambda w(1-\tau_w)z = 0 \tag{19}$$

$$\beta\,\mathrm{E}\left[v_a(a',z')|z\right] - \lambda - \mu - \gamma = 0 \tag{20}$$

$$\mu \leq 0 \tag{21}$$

$$(\underline{a} - a')\mu = 0 \tag{22}$$

$$\gamma \geq 0 \tag{23}$$

$$(\overline{a} - a')\gamma = 0 \tag{24}$$

$$\overline{a} \geq a' \tag{25}$$

plus the two equations (15) and (16). Using an envelope result yields

$$v_a(a,z) = \lambda(1 + (1-\tau_k\mathcal{I}_{a\geq 0})r)\,. \tag{26}$$

Hence, equation (20) becomes

$$\beta\,\mathrm{E}\left[\lambda'(1+(1-\tau_k\mathcal{I}_{a'\geq 0}))r')|z\right] - \lambda - \mu - \gamma = 0 \tag{27}$$

where we have assumed without loss of generality that tax rates stay constant over time. Given the simplification of the notation, the previous expressions are relevant for the steady state and the transition.

A.2 Numerical solution

In general, our numerical procedure aims at precisely calculating the policy functions by iterating on the above first-order conditions and constraints, and then using them to compute the density measure across asset holdings and labor productivity. This method can be used in computing both the steady state and the transition with slight adaptations that will be described below.

We first set up a grid $\mathcal{A}$ on assets with overall negative and positive asset holdings limits $\underline{a}_{\min}$ and $\overline{a}$, and make sure that they are not binding in any of the calibrations. We set these bounds to -20 and 600 and use 250 points for $\mathcal{A}$. The grid oversamples negative holdings: a sampling scheme where about one fifth of the grid points are negative is used. The stochastic process of idiosyncratic productivity is modeled using Rouwenhorst's method (Kopecky and Suen 2010). We use 7 points for $\mathcal{Z}$.

The procedure for calculating the steady state is conditional on some combination of the fiscal variables satisfying the intertemporal government budget described by equations (8)–(10). This means that B, G, Tr, Γ, τ_k and τ_w must satisfy the steady-state relationship

$$\Gamma + \tau_k r \int_{a\geq 0} a\, d\theta + \tau_w w N = G + \mathrm{Tr} + rB\,, \tag{28}$$

knowing that τ_k, τ_w, Tr and G are exogenously given.

There are two levels of iterations. The inner iteration is on individual policy functions. The outer iteration, which will be indexed by i in this section, is on the aggregate prices and quantities.

The measure θ can be defined in a grid considerably finer than $\mathcal{X} = \mathcal{A} \times \mathcal{Z}$ in the first dimension. Without loss of generality, assume a grid $\tilde{\mathcal{X}} = \tilde{\mathcal{A}} \times \mathcal{Z}$, where $\tilde{\mathcal{A}}$ is denser than $\mathcal{A}$ but contains all its elements.[27]

1. Start with a first guess for next period's aggregate capital, value function, autarky value, borrowing limit and consumption policy function, and a first guess for current period's aggregate labor supply, joint distribution of assets and shocks and the asset policy functions,
$$(K^0_{\text{next}}, N^0, v^0_{\text{next}}(a,z), \underline{v}^0_{\text{next}}(z), \underline{a}^0_{\text{next}}, \theta^0, a^0(a,z), c^0_{\text{next}}(a,z))\,.$$
Set the outer iteration index i to 0.

2. Guess the level of debt for the next period and the current level of lump-sum taxes, $(B^0_{\text{next}}, \Gamma^0)$.

3. If solving for the steady state, set $N^i_{\text{next}} = N^i$.

4. Compute prices $(r^i_{\text{next}}, w^i_{\text{next}})$ using (6) and (7). If solving for the steady state, set $r^i = r^i_{\text{next}}$ and $w^i = w^i_{\text{next}}$.

5. Given $\underline{v}^i_{\text{next}}(z)$, compute the autarky value, $\underline{v}^i(z)$ for all values in $\mathcal{Z}$ using (3). The maximization problem in the expression is well-behaved and yields an interior solution in terms of n. Details for computing n are given in step 6.

[27] In the solutions computed in the paper, the two grids are the same because $\mathcal{A}$ already contains a sufficiently large number of points. In even more numerically intensive applications one could decrease the number of points in $\mathcal{A}$ to a much lower number (say, 30) and keep a large number of elements for $\tilde{\mathcal{A}}$.

6. Given policy function $a^i(a,z)$, solve equations (15), (18) and (19) to obtain policy functions
$$\left(n^i(a,z), c^i(a,z), \lambda^i(a,z)\right).$$
The procedure consists of solving (15) and (18) with respect to c and λ, then substituting these variables in the nonlinear equation (19) and solving it with respect to n using the Newton-Raphson method.

7. Using equation (18), compute $\lambda^i_{\text{next}}(a,z) = u_c(c^i_{\text{next}}(a,z),\cdot)$, where the second argument is irrelevant given separability of the arguments of the utility function. Compute
$$\Phi(a,z) = \beta \operatorname{E}\left[\lambda^i_{\text{next}}(a^i(a,z),z')(1+(1-\tau_k \mathcal{I}_{a^i(a,z)\geq 0}))r^i_{\text{next}})|z\right] - \lambda^i(a,z)$$
using linear interpolation where necessary.

8. Use equation (27) to set $\mu^i(a,z) = \Phi(a,z)$ in all points of set $S_1 = \{(a,z) \in \mathcal{X} : a^i(a,z) < \underline{a}^i_{\text{next}} + \epsilon_1\}$, for small $\epsilon_1 > 0$, and zero elsewhere.

9. Use equation (27) to set $\gamma^i(a,z) = \Phi(a,z)$ in all points of set $S_2 = \{(a,z) \in \mathcal{X} : a^i(a,z) > \overline{a}^i_{\text{next}} - \epsilon_2\}$, for small $\epsilon_2 > 0$, and zero elsewhere.

10. Partition grid $\mathcal{X}$ into five mutually exclusive sets:
$$\begin{aligned}
R_1 &= \left\{(a,z) \in S_1 : \mu^i(a,z) > 0\right\} \\
R_2 &= \left\{(a,z) \in S_2 : \gamma^i(a,z) < 0\right\} \\
R_3 &= \left\{(a,z) \in S_1 : \mu^i(a,z) \leq 0\right\} \\
R_4 &= \left\{(a,z) \in S_2 : \gamma^i(a,z) \geq 0\right\} \\
R_5 &= \mathcal{X} \setminus \{R_1 \cup R_2 \cup R_3 \cup R_4\}\ .
\end{aligned}$$
Increase $a^i(a,z)$ in all points of R_1; decrease $a^i(a,z)$ in all points of R_2; set $a^i(a,z)$ equal to $\underline{a}^i_{\text{next}}$ in all points of R_3; set $a^i(a,z)$ equal to $\overline{a}$ in all points of R_4; increase $a^i(a,z)$ in the points of R_5 such that $\Phi(a,z) > 0$; and decrease $a^i(a,z)$ in the points of R_5 such that $\Phi(a,z) < 0$. Set the values of $a^i(a,z)$ larger than $\overline{a}$ (if any) to $\overline{a}$ and the values smaller than $\underline{a}^i_{\text{next}}$ (if any) to $\underline{a}^i_{\text{next}}$.

11. If computing for the steady state, set $c^i_{\text{next}}(a,z) = c^i(a,z)$; compute v^i iterating on expression (14) with policy functions $n^i(a,z)$ and $c^i(a,z)$.

12. Go back to stage 6 until changes in $a^i(a,z)$ in stage 10 are small enough for all points in $\mathcal{X}$; all the above necessary conditions should be satisfied within a small error.

13. Compute the borrowing limit for the current period by solving $v^i(a,z) = \underline{v}^i(z)$ in a with linear interpolation and pick the tightest of these limits, $\underline{a}^i$.

14. Given the policy function $a^i(a,z)$, compute next period's measure $(\theta^i)'$ defined in set $\tilde{\mathcal{X}}$ using linear interpolation to extend the policy function to the denser space. In particular, next period's density in state $(\tilde{a},z) \in \tilde{\mathcal{X}}$ is computed taking into account the distance of next period's asset holdings to the two points of the denser grid. To that effect, define the value in the denser grid of assets $\tilde{\mathcal{A}}$ immediately above the policy function $a^i(\tilde{a},z)$ at a generic point as $a^u(\tilde{a},z) = \max\{\tilde{a} \in \tilde{\mathcal{A}} : a^i(\tilde{a},z) \leq \tilde{a}\}$ and similarly for the value immediately below, $a^l(\tilde{a},z) = \min\{\tilde{a} \in \tilde{\mathcal{A}} : a^i(\tilde{a},z) \geq \tilde{a}\}$. Further define the weights associated to these two points as

$$w^l(\tilde{a},z) = \begin{cases} \frac{a^u(\tilde{a},z) - a(\tilde{a},z)}{a^u(\tilde{a},z) - a^l(\tilde{a},z)} & \text{if } a^u(\tilde{a},z) \neq a^l(\tilde{a},z) \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

and $w^u(\tilde{a},z) = 1 - w^l(\tilde{a},z)$.

The next period measure can be computed from the current period measure with generic density $\theta^i_{(\tilde{a},z)}$ as:

$$\left(\theta^i\right)'_{(\tilde{a}',z')} = \sum_{(\tilde{a},z)\in H^u(\tilde{a}')\times\mathcal{Z}} \Pi(z,z')\theta^i_{(\tilde{a},z)} w^u(\tilde{a},z) + \sum_{(\tilde{a},z)\in H^l(\tilde{a}')\times\mathcal{Z}} \Pi(z,z')\theta^i_{(\tilde{a},z)} w^l(\tilde{a},z)$$

where $H^u(\tilde{a}') = \{(\tilde{a},z) \in \tilde{\mathcal{X}} : a^u(\tilde{a},z) = \tilde{a}'\}$ and $H^l(\tilde{a}') = \{(\tilde{a},z) \in \tilde{\mathcal{X}} : a^l(\tilde{a},z) = \tilde{a}'\}$.

15. Compute the desired level of capital in the next period and the supply of labor in the current period given by expressions (11) and (12), whose discrete counterparts are:

$$K^i_s = \sum_{\tilde{\mathcal{X}}} a^i(\tilde{a},z)\theta^i_{(\tilde{a},z)} - B^i_{\text{next}}$$
$$N^i_s = \sum_{\tilde{\mathcal{X}}} n^i(\tilde{a},z)\theta^i_{(\tilde{a},z)} .$$

16. Set B^{i+1}_{next} and Γ^{i+1} using equations (8)–(10).

17. If $K^i_{\text{next}} - K^i_s$, $N^i - N^i_s$, $B^{i+1}_{\text{next}} - B^i_{\text{next}}$ and $\Gamma^{i+1} - \Gamma^i$ are small enough in absolute terms, one should have a solution for the problem and stop here.

18. Otherwise, set K^{i+1}_{next} to a number between K^i_s and K^i_{next}, and similarly for N^{i+1}. Set θ^{i+1} to $(\theta^i)'$, $a^{i+1}(a,z)$ to $a^i(a,z)$, $c^{i+1}_{\text{next}}(a,z)$ to $c^i(a,z)$, $\underline{a}^{i+1}_{\text{next}}$ to $\underline{a}^i$, $\underline{v}^{i+1}_{\text{next}}(z)$ to $\underline{v}^i(z)$ and $v^{i+1}_{\text{next}}(a,z)$ to $v^i(a,z)$. Increment i by one and go back to stage 3.

Regarding the transition, we point out the following. The transition exercise consists of calculating the evolution of the economy starting with a certain distribution of assets and shocks θ^{Init} and the paths for the exogenous quantities, like transfers or government spending.

Set the simulation horizon, T, to a large number, say 600 periods. Instead of guesses for aggregate capital, labor supply, the policy and value functions, the autarky value, the fiscal variables, the joint distribution of assets and shocks, and the borrowing limit, we need to have a first guess for the entire path of those quantities. In practical terms, a good first guess for the paths of these quantities is, for all T periods, their values at the final steady state.

We then have to proceed in the following way. Identify the iteration label i with the time period $t+1$ and the results for the next iteration, denoted by $i+1$, with time t. The idea is to start from the end of the horizon and recursively proceed to the initial period. We start in moment t equal to $T-1$, so that i is 0. Run steps 6–12 above using the same computational routines as for the steady state. Then, update t to $T-2$ and repeat this cycle until t is 1. The next part of the problem is to update the distributions of assets and shocks given the policy functions just calculated. Use the distribution θ^{Init} and the policy functions to update the entire path of the joint distribution and the other macro level variables, including the borrowing limit.

For each of the T periods compare the computed aggregate capital with the guess, and likewise for aggregate labor, following the general idea of stage 17. Repeat the entire procedure until the differences between the computed aggregate capital and aggregate labor and their guesses are sufficiently small in all periods, and the changes in the endogenous fiscal variables between iterations are also small enough in all periods.

B Policy functions in the steady state

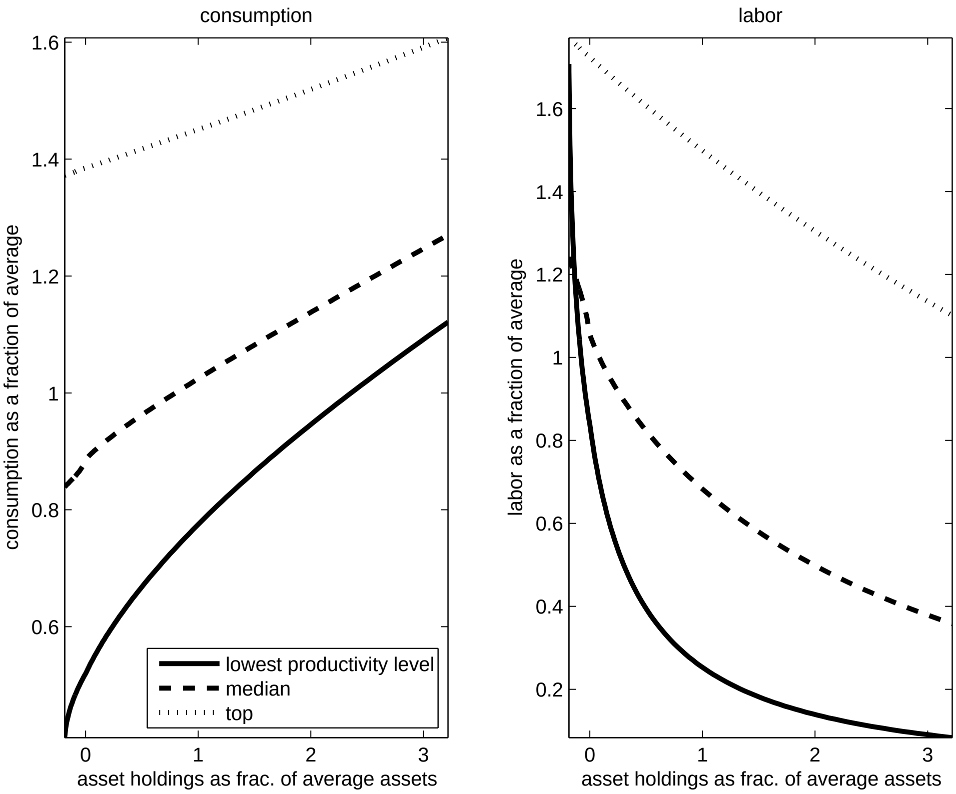

Figure 6: Consumption and labor policy functions in steady state, conditional on different levels of idiosyncratic productivity.

Figure 6 shows the agents' policy functions associated with consumption and labor for the lowest, the median and the highest productivity level. Focusing on the consumption policy function parameterized in the lowest productivity level, we see that it exhibits more curvature as wealth approaches the borrowing limit. This is typical in models with precautionary saving motives and borrowing limits (Zeldes 1989, Carroll and Kimball 1996). As expected, the curvature diminishes as the level of the idiosyncratic productivity increases. The labor policy functions mirror those of consumption; more specifically, borrowers and wealth-poor households at the lowest level of productivity are the most responsive in terms of labor. A marginal increase in their wealth produces a negative and large reaction in their labor supply.

C Transition: additional results for the debt-financed transfers policy

This section shows additional simulations together with several robustness exercises associated with the debt-financed transfers policy.

C.1 The anatomy of the tightening and its effects on the policy functions

Figure 7 shows the movement of the borrowing constraint as a result of the reaction of the equilibrium and autarky value functions (parameterized in the lowest z) to the implemented policy. Specifically, we show the movement of this constraint from its steady-state position (vertical solid line) to the new position in the first relevant period after the policy change (vertical dotted line).[28]

Because of the transfers policy, both value functions $\upsilon(x', \theta')$ and $\underline{\upsilon}(z', \theta')$ move up. Crucially, the increase in the borrowing cost affects (negatively) only the dynamics of $\upsilon(x', \theta')$ which, indeed, moves up by less than what $\underline{\upsilon}(z', \theta')$ does. This fact generates a movement of the borrowing constraint towards zero, or equivalently, a shift of the limit to the right.

Let us mention that the initial unexpected shift of the borrowing limit can in principle leave some households out of the capital grid given that capital is predetermined. Dealing with this issue would require some form of bankruptcy in equilibrium which, as already indicated, is a circumstance not considered in our model. However, right after the fiscal policy is implemented all households will perfectly know the dynamic of the borrowing limit, hence ensuring that their optimal asset choice is feasible.

Figure 8 compares policy functions in the baseline model and in the fixed constraint economy. Specifically, each line reports the difference between the labor, consumption and asset policy functions in the baseline model, and the respective policy functions in the fixed constraint model, for different levels of productivity. These values are calculated at the moment of the policy implementation, that is, at $t = 1$.

[28]The first relevant period is $t = 2$ because it is the moment when the maximum amount of borrowing relevant for the asset holdings' decision at $t = 1$ (the policy implementation date) is defined.

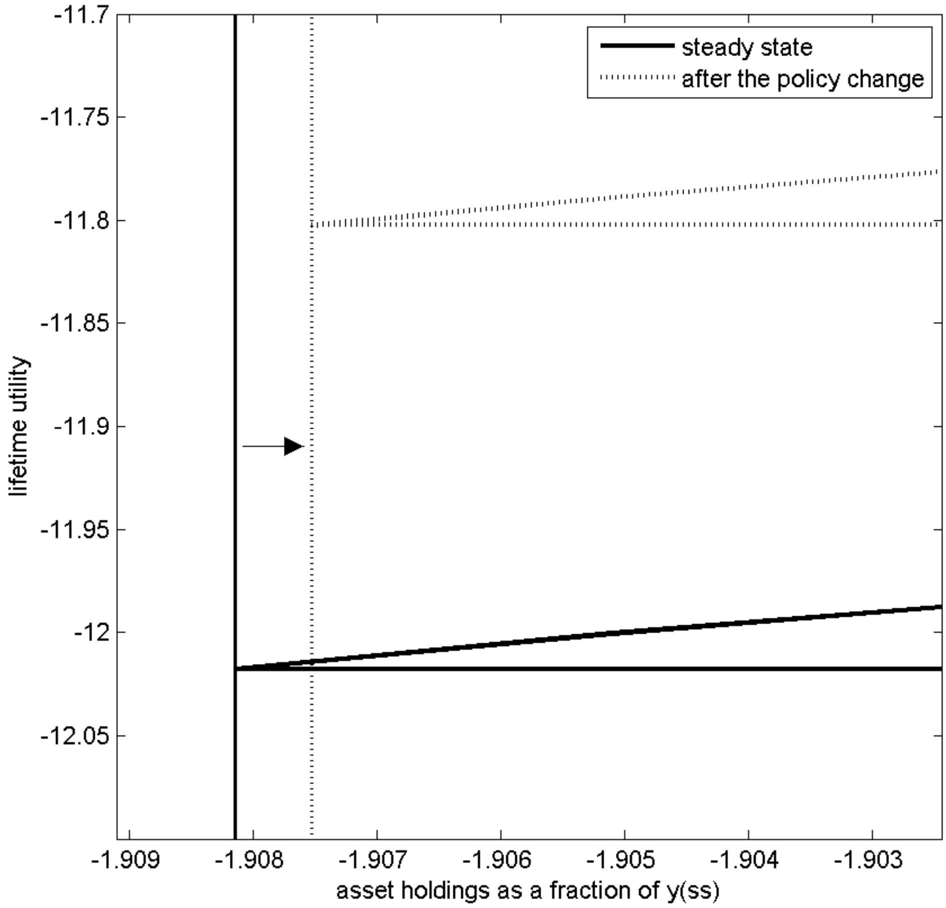

Figure 7: The movement of the borrowing constraint, $\underline{a}(\theta')$, conditional on the implementation of the transfers policy. The two flat lines correspond to the value functions in autarky, $\underline{v}(z', \theta')$, in the steady state (solid line) and in the first relevant period after the policy change (dotted line). The two lines with positive slope refer to the equilibrium value functions, $v(x', \theta')$, in the steady state (solid line) and in the first relevant period after the policy change (dotted line). The value functions are parameterized in the lowest productivity level. The steady-state borrowing limit is identified by the vertical solid line, while the borrowing limit after the change is represented by the vertical dotted line. y(ss) stands for steady-state output.

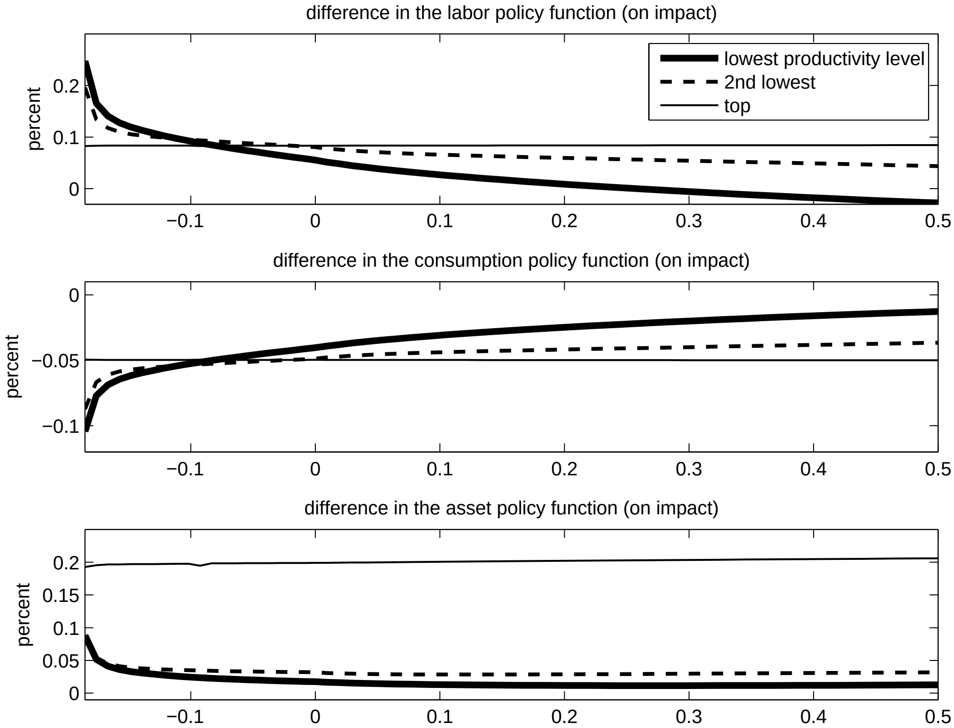

Figure 8: Effects of the transfers policy on labor, consumption and asset decision rules, conditional on different levels of productivity. Each line shows the difference between the policy function obtained in the baseline model and that obtained within the fixed constraint model, for a given productivity level, calculated at $t = 1$. The x-axes are asset holdings as a fraction of average assets.

The three decision rules differ across the two models. In particular, the resulting information is consistent with the households' reactions presented in the main text, that is, households work (consume) on average more (less) because of the tightening. They also tend to increase their wealth. However, for low productivity levels these differences are larger in absolute value as wealth approaches the borrowing limit. In contrast, for the highest productivity level these differences are pretty homogenous across wealth levels.

C.2 Robustness exercises on the dynamics of the borrowing constraint

We now present a number of robustness simulations targeted to the dynamics of the borrowing constraint.

Results with fixed prices We want to study the importance of the dynamics of the interest rate for the occurrence of the tightening. Figure 9 presents the effects of the transfers policy on the borrowing limit, within a fixed prices (off-equilibrium) version of the model. Specifically, we simulate the model while keeping the interest rate at its steady-state value throughout the whole transition.[29] Because of this, after some quarters, the equilibrium value function moves down by less than the autarky value function. Therefore, after an initial mild tightening, the

[29]Notice that keeping the interest rate constant implies also a constant wage rate. In practice, we iterate on policy functions conditional on factor prices kept at their steady-state levels.

borrowing limit loosens persistently.[30]

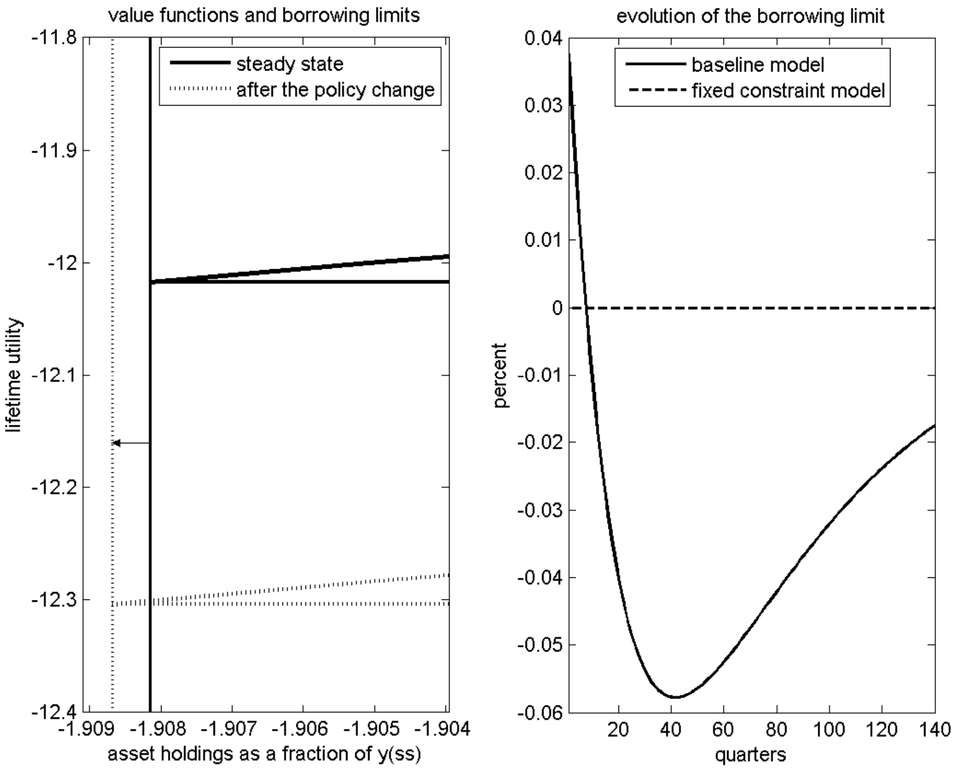

Figure 9: The movement of the borrowing constraint, $\underline{a}(\theta')$, conditional on the implementation of the transfers policy, under fixed prices. In the left panel, the two flat lines correspond to the value functions in autarky, $\underline{v}(z', \theta')$, in the steady state (solid line) and 30 quarters after the policy change (dotted line). The two lines with positive slope refer to the equilibrium value functions, $v(x', \theta')$, in the steady state (solid line) and 30 quarters after the policy change (dotted line). The value functions are parameterized in the lowest productivity level. The steady-state borrowing limit is identified by the vertical solid line, while the borrowing limit 30 quarters after the policy change is represented by the vertical dotted line. In the right panel, the solid line corresponds to the evolution of the borrowing limit over time, while the dashed line identifies the constraint at its steady-state level. The borrowing limit is expressed in deviation from the steady-state level and normalized by steady-state output, y(ss).

The endogenous ad hoc borrowing limit We simulate the debt-financed transfers policy using a borrowing limit which represents a modified version of the natural borrowing limit in Aiyagari (1994). Specifically, we let the borrowing limit be $\underline{a} = -\eta \frac{w}{r}$, where η is parameterized so as to match the credit-to-output ratio in the data.[31] The steady-state value of this alternative borrowing limit is the same we had using the original baseline model. Figure 10 shows that a persistent tightening obtains after a few quarters. Initially, the borrowing constraint loosens because it is directly affected by fall in the interest rate on impact. After some quarters, the limit significantly tightens before slowly coming back to its steady-state level. The magnitude

[30]Notice that in the left panel of Figure 9 we present the value functions at $t = 30$ so as to emphasize the behavior of the value functions when the limit loosens.

[31]We recall that in the standard version of the natural borrowing limit η would be the lowest level of the idiosyncratic productivity.

of the tightening obtained under this ad hoc specification for the borrowing limit is larger than under the rational constraint described in Section 2.

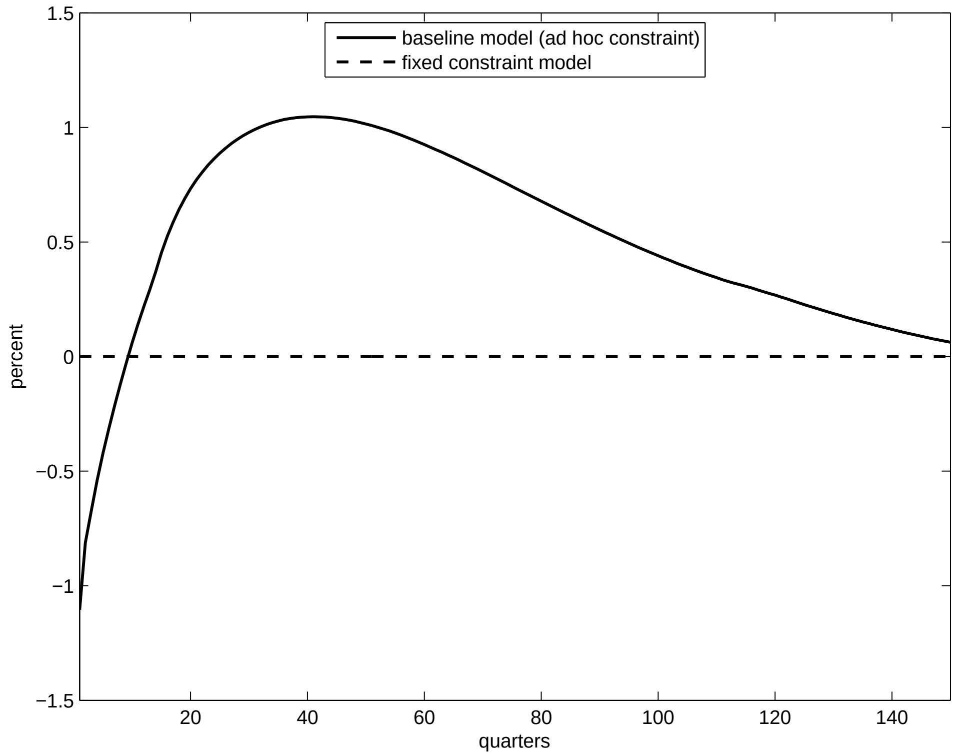

Figure 10: Evolution of the alternative borrowing limit, $-\eta\frac{w}{r}$, under the transfers policy. The borrowing limit is expressed in deviation from the steady-state level and normalized by steady-state output.

Different steady-state levels of public debt In the baseline simulations we start from a steady state of zero public debt. We recalibrate our economy and produce two additional steady states, one with a debt-to-output ratio of 60% (in yearly terms), and the other with 120%.[32] Figure 11 shows the evolution of the borrowing constraint under the transfers policy starting from a steady state of 60% for the debt-to-output ratio; this dynamics is similar to that obtained using the original calibration. The simulation in the case of a steady state of 120% for the debt-to-output ratio delivers similar results, which are available upon request.

Additional exercises We simulate our model using a different value for the parameter ϕ in the fiscal rule; we use the value proposed by Uhlig (2010) of 0.05. Furthermore, we simulate different profiles for the the policy, including a single-period increase in transfers of 1% of steady-state output. Finally, we simulate the model using labor income taxes to finance the policy, instead of lump-sum taxes. All these simulations produce a tightening. The results are available upon request.

[32]In these new steady states we have the same tax rates and lump-sum taxation of the original calibration, but we set a slightly lower level of government purchases so that the fiscal primary balance is consistent with the chosen level of debt.

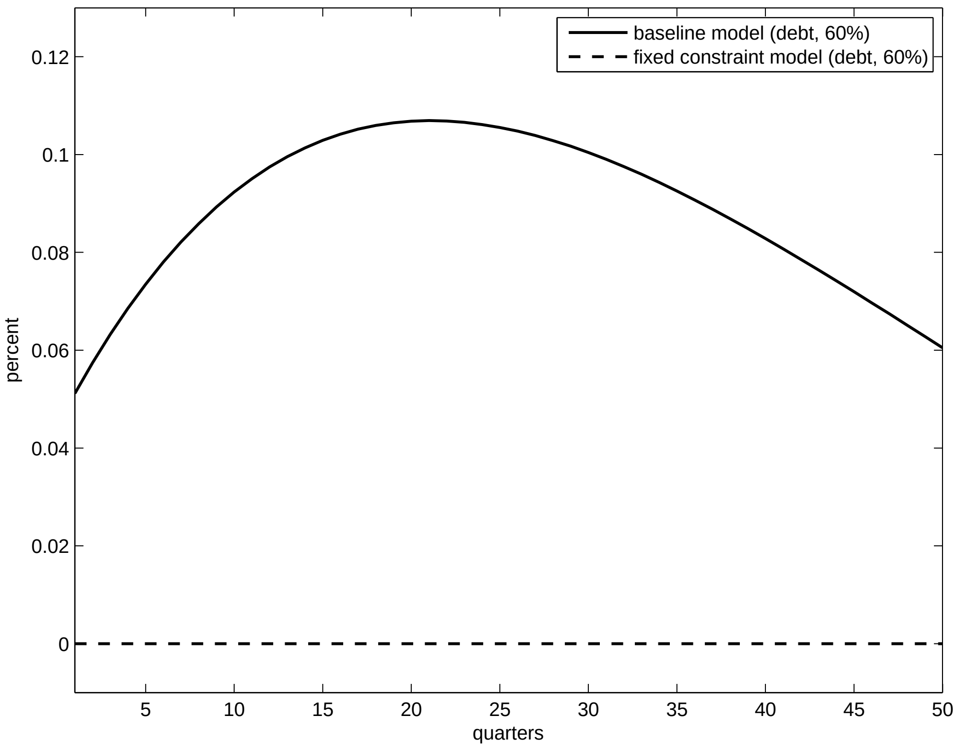

Figure 11: Evolution of the borrowing limit conditional on a steady-state level of public debt-to-output ratio of 60%, under the transfers policy. The borrowing limit is expressed in deviation from the steady-state level and normalized by steady-state output.

C.3 Robustness exercises on the aggregate effects generated by the dynamics of the borrowing constraint

Following the exercises in Section C.2, we check what are the effects produced by the tightening if (i) we use the alternative borrowing limit (the endogenous ad hoc borrowing limit), and (ii) we start from a level of 60% of debt-to-output ratio.

Regarding (i), Figure 12 shows the comparison between selected reactions produced by the model with the endogenous ad hoc borrowing limit and those generated by the fixed constraint model. The effects generated by the tightening in Figure 12 are larger than those obtained in our baseline simulations using the rational borrowing constraints. This suggests that the effects generated by the dynamics of the household borrowing constraint are dampened if the borrowing limit depends not only on prices but also on the value functions, which are directly affected by the future path of all the model's variables.

Regarding (ii), Figure 13 shows the effects generated by the tightening within an economy with 60% level of debt-to-output ratio in the steady state, under the transfers policy. These effects have a similar magnitude to those produced in the context where the steady-state level of public debt is nil.

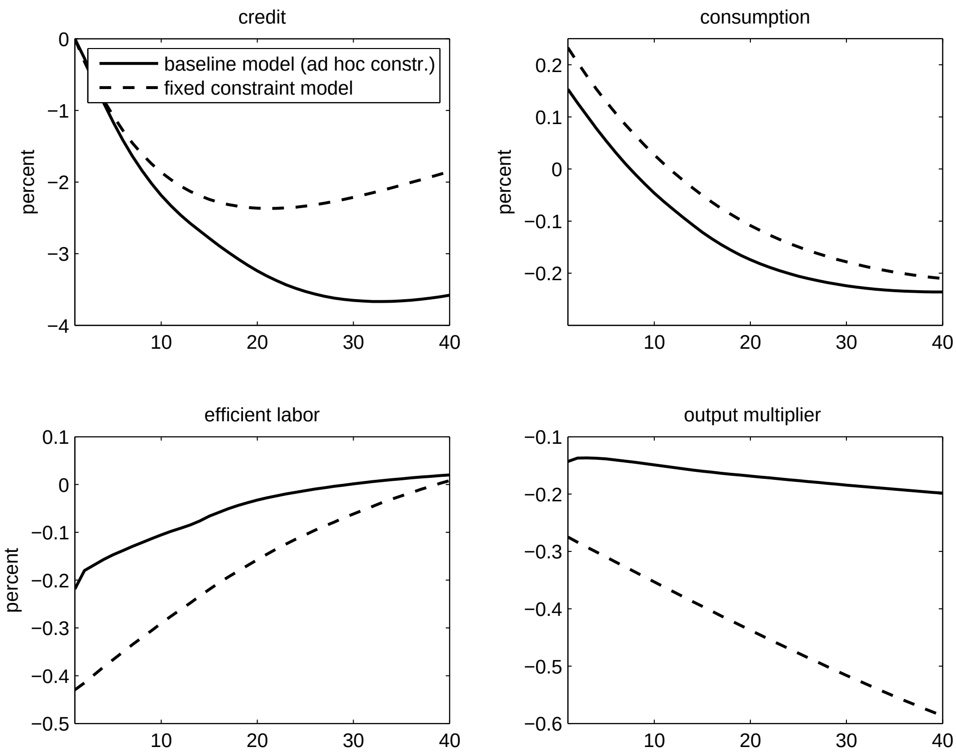

Figure 12: Aggregate effects of the transfers policy under the alternative borrowing constraint. Solid lines are generated with the model that uses the alternative borrowing limit, $-\eta\frac{w}{r}$; dashed lines are generated with the fixed constraint model. All variables are expressed in deviation from steady-state levels. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axes are in quarters.

D Transition: results for the debt-financed purchases policy

This section presents the results associated with the government debt expansion that finances an increase in purchases.

D.1 Understanding the dynamics of the borrowing constraint

The purchases policy We simulate a debt expansion that finances an increase in purchases (G) similar to that of the ARRA. We borrow the process for government spending from Uhlig (2010). On impact, the stimulus amounts to around 0.3% of output, reaching its maximum (around 0.8% of output) after 6–7 quarters.[33] As in the transfers policy, we set the parameter ϕ in the fiscal rule (10) to 0.02. The two top panels in Figure 14 show the evolution of the fiscal variables.

The process of the tightening Figure 14 presents selected reactions to the purchases policy. As stated in the main text, solid lines are generated within our baseline model, where

[33]Notice that process in Uhlig (2010) is characterized by a zero increase of G on impact, and a 0.3% of output increase in the second period. We start in the second period of that process in order to avoid unbounded multipliers on impact. The G path follows an AR(2) process, with the coefficients on the first and the second autoregressive terms being equal to 1.653 and -0.672, respectively.

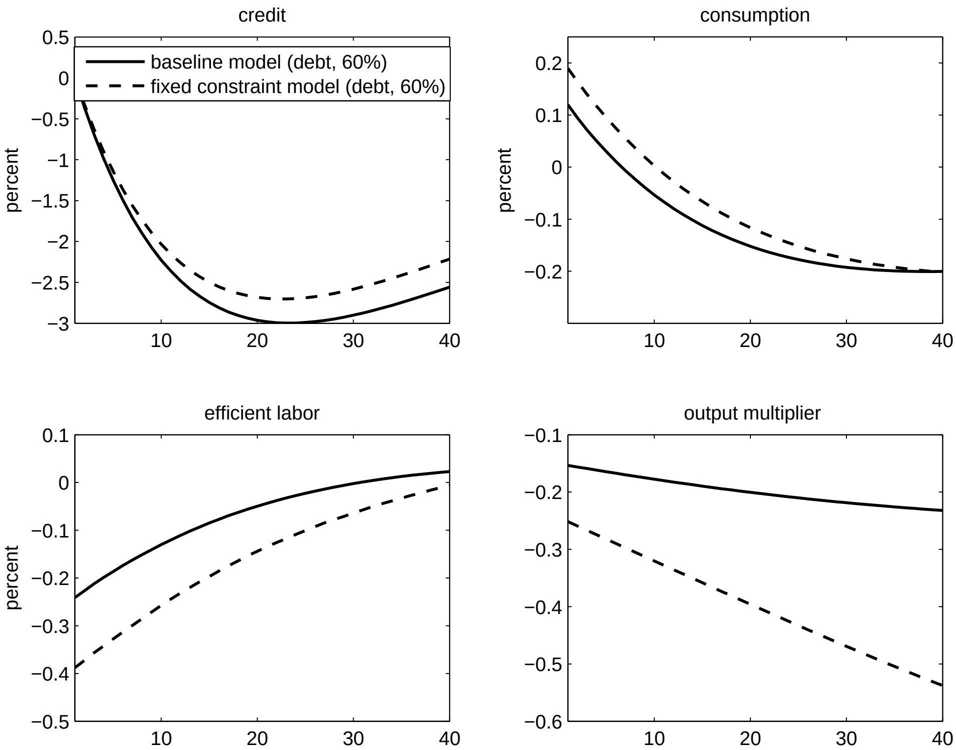

Figure 13: Aggregate effects of the transfers policy, conditional on a steady-state level of public debt-to-output ratio of 60%, in yearly terms. Solid lines are generated with the baseline model and dashed lines with the fixed constraint model. Both model versions feature a steady-state level of public debt-to-output ratio of 60%, in yearly terms. All variables are expressed in deviation from steady-state levels. The cumulative dynamic multipliers are calculated following Uhlig (2010). The x-axes are in quarters.

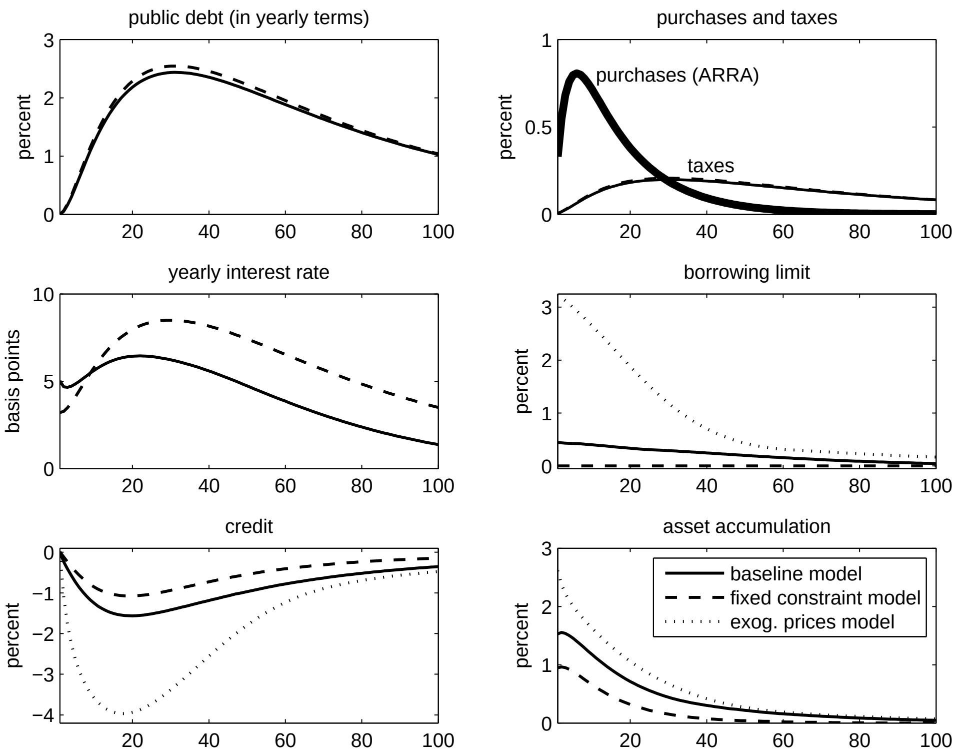

Figure 14: Selected aggregate effects of the purchases policy. All variables are expressed in deviation from steady-state levels. The deviations for public debt, transfers, taxes and the borrowing limit are normalized by steady-state output. Positive deviations for the borrowing limit mean a tightening. Asset accumulation is the difference between the asset level in $t+1$ and the steady-state level, normalized by steady-state investment. The x-axes are in quarters.

the borrowing limit is allowed to evolve endogenously. Dashed lines, instead, pertain to the fixed constraint model. Dotted lines identify the exogenous prices model (see Section 4.1 for a detailed description of this case).

The effects of the policy on the borrowing constraint are similar to those obtained with the transfers policy. That is, issuing public debt positively impacts the interest rate. All else equal, this creates a tightening of the borrowing constraint because the option of staying in the market becomes relatively worse than going to autarky. The tightening is persistent over time, and stimulates demand for assets that generates a downward pressure on the interest rate. Indeed, the interest rate is on average lower in the baseline model than in the fixed constraint model. The highest fall in credit is obtained in the exogenous prices model, followed by that observed in the baseline model, and finally by that of the fixed constraint model. The behavior of asset accumulation mirrors that of credit.

Quantitatively, the change in the borrowing limit in both the exogenous prices and the baseline versions are larger than under the transfers policy. This can be due to the fact that the increase in the interest rate is higher under the purchases policy. It can also be a consequence of the completely different nature of the purchase policy vis-à-vis the transfers policy (more details on this below).

D.2 The anatomy of the tightening and its effects on the policy functions

The purchases policy is fundamentally different from the transfers policy. The former creates the so-called negative wealth effect in the economy, which, on average, induces agents to work more and consume less. Contrary to the transfers policy, Figure 15 shows that the purchases policy makes both the autarky and equilibrium value functions fall. Similarly to the transfers policy, the tightening eventually occurs because the increase in the borrowing cost affects only the dynamics of $v(x', \theta')$, which moves down more than what $\underline{v}(z', \theta')$ does.

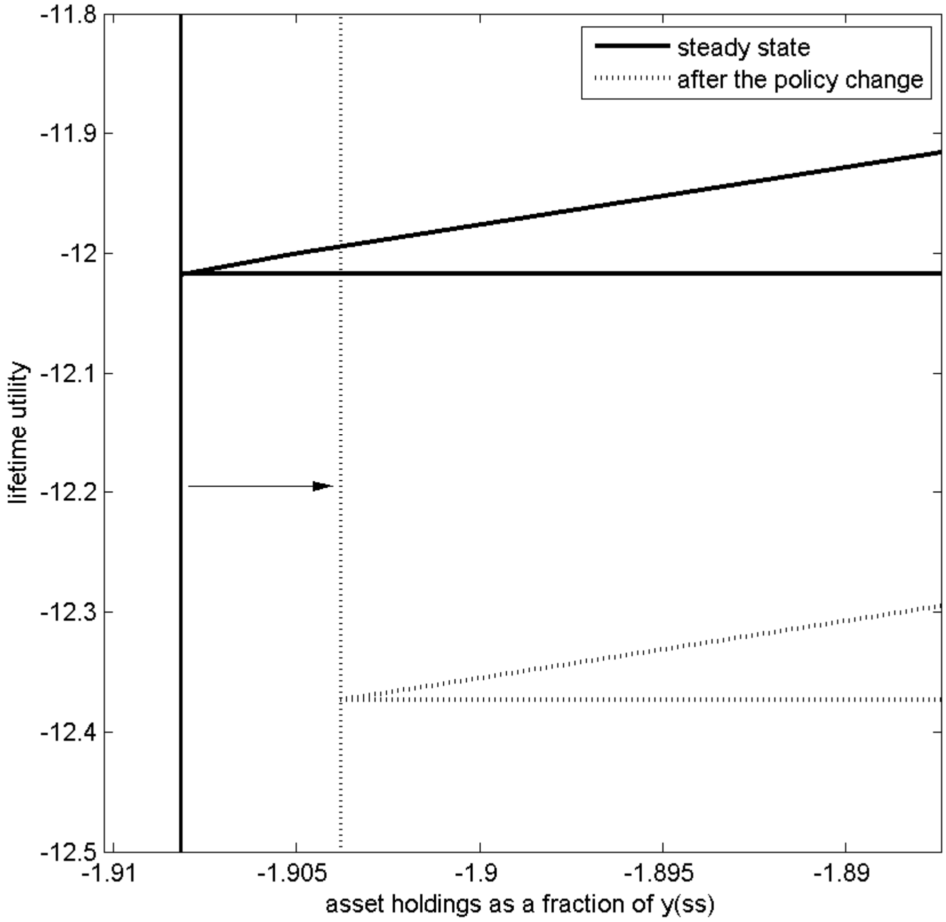

Figure 15: The movement of the borrowing constraint, $\underline{a}(\theta')$, conditional on the implementation of the purchases policy. The two flat lines correspond to the value functions in autarky, $\underline{v}(z', \theta')$, in the steady state (solid line) and in the first relevant period after the policy change (dotted line). The two lines with positive slope refer to the equilibrium value functions, $v(x', \theta')$, in the steady state (solid line) and in the first relevant period after the policy change (dotted line). The value functions are parameterized in the lowest productivity level. The steady-state borrowing limit is identified by the vertical solid line, while the borrowing limit after the change is represented by the vertical dotted line. y(ss) stands for steady-state output.

Figure 16 compares policy functions in the baseline model and in the fixed constraint economy. The meaning of each line is explained in Section C.1. As in the transfers case, we note that the three decision rules are different across the two models and that these differences vary along the grid of assets for low productivity levels. Consistently with the results in Section D.1, the effects of the tightening on the policy functions are larger than those under the transfers policy.

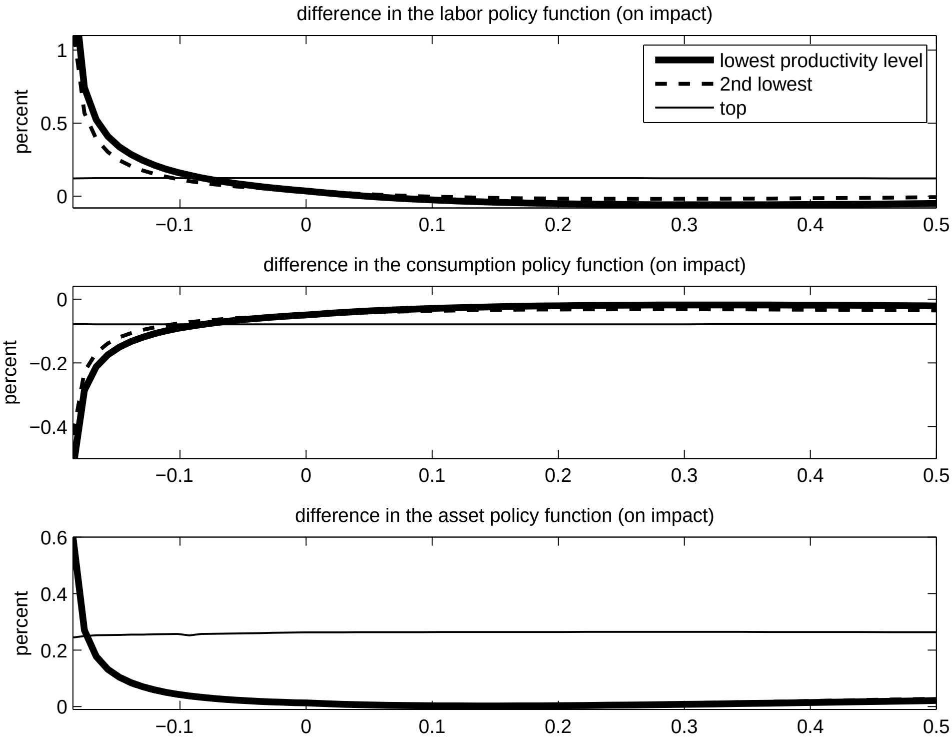

Figure 16: Effects of the purchases policy on labor, consumption and asset decision rules, conditional on different levels of productivity. Each line shows the difference between the policy function obtained in the baseline model and that obtained within the fixed constraint model, for a given productivity level, calculated at $t = 1$. The x-axes are asset holdings as a fraction of average assets.

D.3 Dynamics at the individual and aggregate level

Figures 17 and 18 show the heterogeneous and aggregate reactions to the policy. The households' reactions generated by the tightening under the purchases policy are qualitatively similar to those generated under the transfers policy. However, under the purchases policy, the price effects work differently in that they dampen the responses obtained in the exogenous price economy for all households. The effects of the borrowing limit (depurated from its prices effects) on constrained households' reactions are larger than under the transfers policy. All these facts have an implication for the output effects of the purchases policy: the impact output multiplier in the exogenous prices economy is around 2 while it is around 1 in the baseline model.

D.4 Welfare

Figure 19 presents the welfare gains along the household wealth distribution for the baseline (solid line) and the fixed constraint model (dashed line), under the purchases policy. To assess the welfare gains we follow the procedure described in Section 4.3.

Within the fixed constraint model, the government spending policy produces negative effects for almost all households along the wealth distribution. Only the richest households (top 10%)

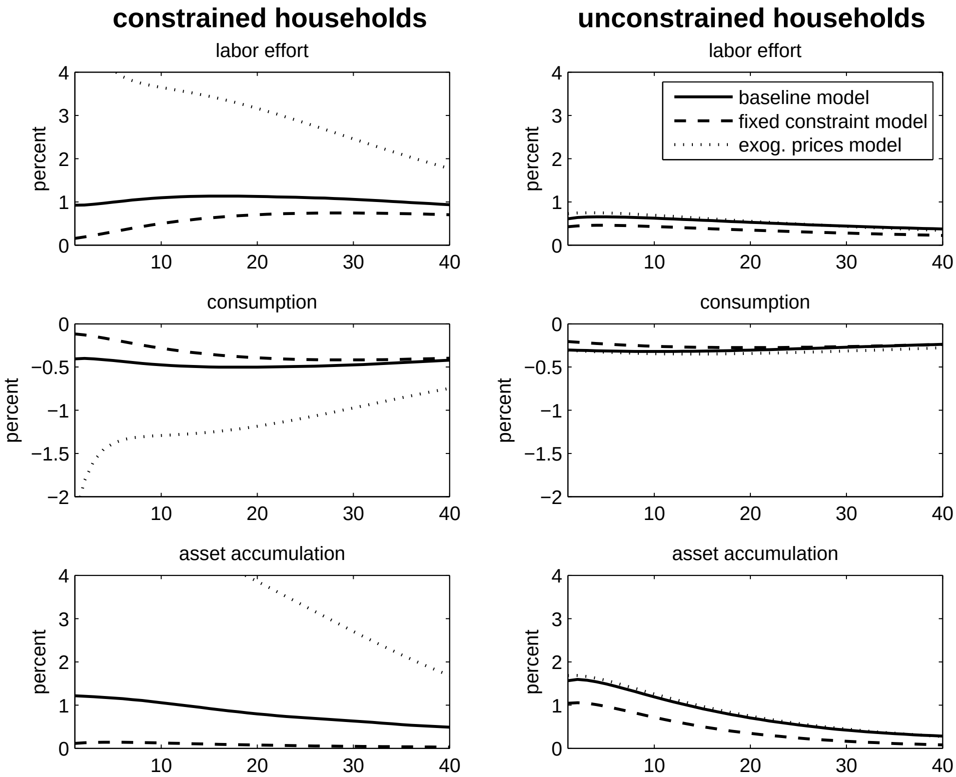

Figure 17: Heterogeneous effects of the purchases policy. The average reactions of constrained agents are on the left column. The average reactions of unconstrained agents are on the right column. All variables are expressed in deviation from steady-state levels. Asset accumulation is the difference between the asset level in $t+1$ and the steady-state level, normalized by steady-state investment. The x-axes are in quarters.

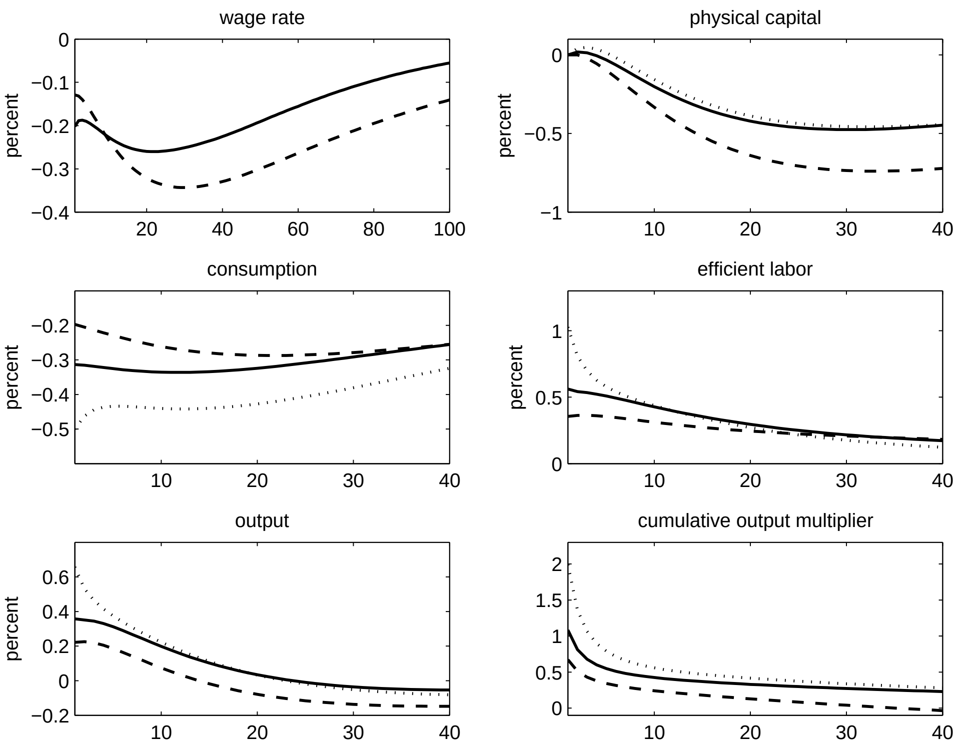

Figure 18: Aggregate effects of the purchases policy. All variables are expressed in deviation from steady-state levels. The cumulative output multiplier is calculated following Uhlig (2010); see footnote 21. The x-axes are in quarters.

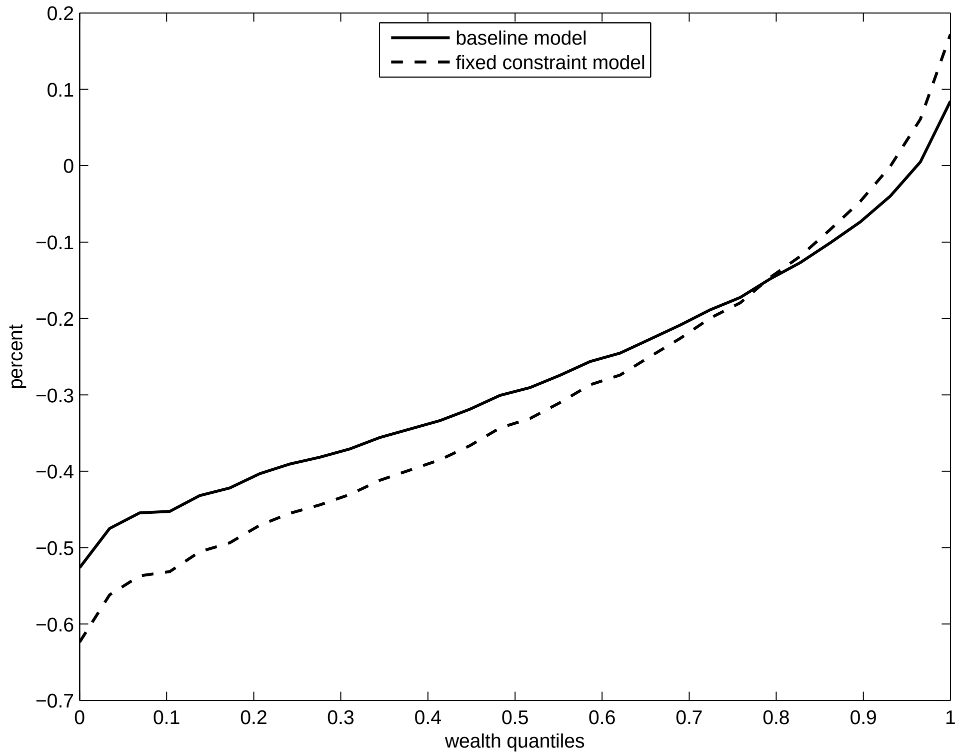

Figure 19: Welfare effects of the purchases policy by wealth quantiles. Positive values represent a welfare gain measured as the consumption-equivalent change relative to the initial steady state. See the definition in footnote 24.

experience a welfare gain, possibly because the higher yield more than compensates the negative effects coming from the future higher taxation.

The welfare associated to the baseline model is higher than that of the fixed constraint model for all households up until roughly the fourth quintile. Instead, for richer households the opposite holds. As in the transfers case, we explain this fact relying on the price effects generated by the tightening: relative higher (lower) wage (interest) rates which, on the one hand, favor the constrained, the wealth-poor and the middle class households, and, on the other hand, disappoint the richest households.

D.5 Robustness exercises

We present here the results with fixed prices and list a set of additional robustness exercises that were also performed.

Results with fixed prices As in the transfers case, we simulate a fixed prices version of the model in which the interest rate is kept fixed at its steady-state value throughout the whole transition. Figure 20 shows the dynamics of the borrowing limit in that case. Conditional on the implementation of the purchases policy, the borrowing constraint becomes looser with respect to its steady-state value in any point of the transition. Again, the role of price dynamics in the baseline model version is crucial for generating the tightening.

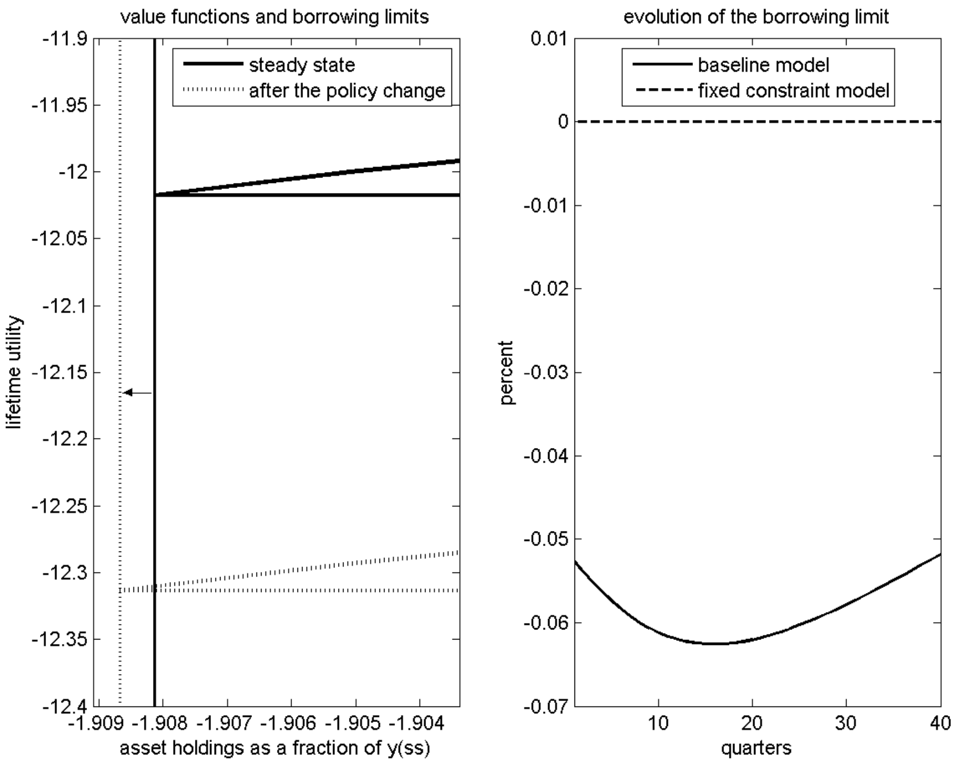

Figure 20: The movement of the borrowing constraint, $\underline{a}(\theta')$, conditional on the implementation of the purchases policy, under fixed prices. In the left panel, the two flat lines corresponds to the value functions in autarky, $\underline{v}(z', \theta')$, in the steady state (solid line) and in the first relevant period after the policy change (dotted line). The two lines with positive slope refer to the equilibrium value functions, $v(x', \theta')$, in the steady state (solid line) and in the first relevant period after the policy change (dotted line). The value functions are parameterized in the lowest productivity level. The steady-state borrowing limit is identified by the vertical solid line, while the borrowing limit after the change is represented by the vertical dotted line. In the right panel, the solid line corresponds to the evolution of the borrowing limit over time, while the dashed line identifies the constraint at its steady-state level. The borrowing limit is expressed in deviation from the steady-state level and normalized by steady-state output, y(ss).

Other robustness exercises We performed the same set of robustness exercises as in Sections C.2 and C.3. As in the case of the transfers policy, the robustness exercises under the purchases policy deliver results which are similar to those obtained under the baseline model. For ease of space, we omit the presentation of these results which are available upon request.

E A crisis experiment

The purpose of this section is to see how public debt expansions affect the agents' behavior during an economic crisis characterized by both a fall in output and in private credit.

In order to generate a crisis scenario, we simulate a negative technology change (or a decrease in TFP), modeled as fall in A as defined in Section 2.1. In particular, A follows the deterministic process $A' = 1-\rho^a+\rho^a A$, where we set the initial level of A to 0.965 and $\rho^a = 0.85$.[34] Within our baseline model specification, we unexpectedly implement the mentioned deterministic process for A at $t = 1$. Solid lines in Figure 21 show selected reactions to this technology change. All else equal, the marginal product of capital decreases; the real interest rate and the capital stock fall. Households consume and work less. An initial fall in output of roughly 4% is generated. On impact, a tightening of the borrowing constraint obtains of about 5% of steady-state output. The prices dynamics contributes to the tightening: the borrowing cost, though initially decreasing, is higher than its steady-state level after roughly two years and remains positive for more than ten years.[35] The tightening favors a fall in credit.

The effects generated by the TFP fall alone can be seen as a "no intervention" scenario where the fiscal authority does not implement any discretionary policy. In order to see how the crisis is affected by the fiscal policies we proceed as follows. At the same time of the occurrence of the technology change, we simulate, in turn, the two debt expansions described in Sections 4.1 and D.1 of the appendix. The dashed lines represent the variables' reactions to the TFP fall plus the debt-financed transfers policy. The dotted lines represent the variables' reactions to the TFP fall plus the debt-financed purchase policy. Though the quantitative effects of the two fiscal policies are somehow different, the qualitative effects show several similarities. For example, both policies produce a higher level of the borrowing cost when compared to the no intervention scenario. This is consistent with the fact that the implementation of the policies contributes to a further tightening of the borrowing limit. Both policies generate a fall in credit: five years after the start of the policies, the fall in credit is, on average, twice as large as that obtained in the no intervention case. Furthermore, because of the behavior of the interest rate, physical capital is crowded out more if the fiscal policies are implemented. Regarding the consumption dynamics, only the transfers policy alleviates mildly the impact fall in consumption generated by the decrease in TFP. Finally, the effects of the policies on output

[34] We set the initial value of A in order to roughly get a 4% fall in output. Such a fall is the same observed for the U.S. GDP during the Great Recession, specifically, between 2008:Q2 and 2009:Q2. Regarding ρ^a, we use different values around the chosen one; results hardly change.

[35] Because of the TFP fall, the equilibrium and the autarky value functions (parameterized in the lowest z) fall; however, the equilibrium value function falls more than the autarky one because of the dynamics of the borrowing cost. This produces a reduction of the maximum amount of borrowing.

are modest; in particular, the implementation of the purchases policy alleviates the recession by less than 0.5% of output on impact.

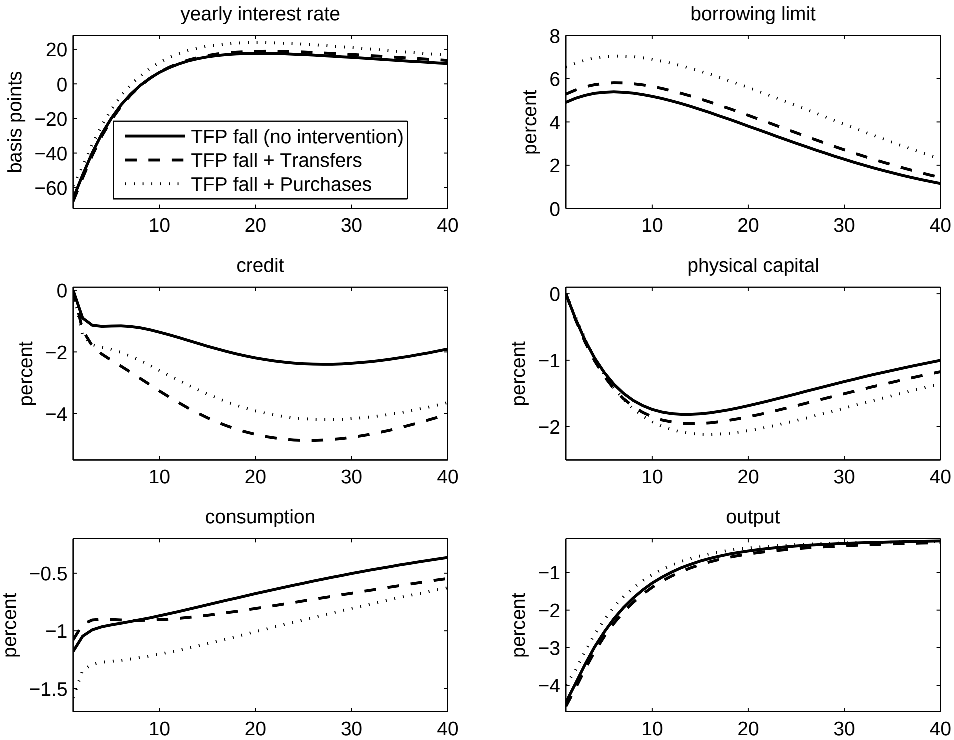

Figure 21: Selected aggregate effects in a crisis experiment. All reactions are generated using the baseline version of the model. Solid lines represent the reactions to a TFP fall without any intervention from the fiscal authority. Dashed lines are generated by the occurrence of both the TFP fall and the transfers policy. Dotted lines are generated by the occurrence of both the TFP fall and the purchases policy. All variables are expressed in deviation from steady-state levels. The deviations for the borrowing limit are normalized by steady-state output. The x-axes are in quarters.